Calhoun: The NPS Institutional Archive
DSpace Repository

Theses and Dissertations 1. Thesis and Dissertation Collection, all items

1962

Effect of longitudinal heat conduction on rotary regenerators.

Bahnke, Gustavo D.

Monterey, California: U.S. Naval Postgraduate School

http://hdl.handle.net/10945/12056

Downloaded from NPS Archive: Calhoun

Calhoun is the Naval Postgraduate School's public access digital repository for research materials and institutional publications created by the NPS community. Calhoun is named for Professor of Mathematics Guy K. Calhoun, NPS's first appointed -- and published -- scholarly author.

Dudley Knox Library / Naval Postgraduate School
411 Dyer Road / 1 University Circle
Monterey, California USA 93943

http://www.nps.edu/library

NPS ARCHIVE
1962
BAHNKE, G.

AD-480916

EFFECT OF LONGITUDINAL HEAT CONDUCTION ON ROTARY REGENERATORS

GUSTAVO D. BAHNKE

LIBRARY
U.S. NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA

[redacted]

UNCLASSIFIED

[redacted]

EFFECT OF LONGITUDINAL HEAT CONDUCTION
ON ROTARY REGENERATORS

* * * * *

Gustavo D. Bahnke

[redacted]

UNCLASSIFIED

[redacted]

[redacted]

EFFECT OF LONGITUDINAL HEAT CONDUCTION ON ROTARY REGENERATORS

by

Gustavo D. Bahnke

Lieutenant, Chilean Navy

Submitted in partial fulfillment of
the requirements for the degree of
MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California
1962

NPS ARCHIVE
1967
Bahnke, G

~~Thesis
B34~~

LIBRARY
U.S. NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA

EFFECT OF LONGITUDINAL HEAT CONDUCTION ON ROTARY REGENERATORS

by

Gustavo D. Bahnke

This work is accepted as fulfilling
the thesis requirements for the degree of

MASTER OF SCIENCE

IN

MECHANICAL ENGINEERING

from the

United States Naval Postgraduate School

ABSTRACT

The general differential equations describing the behavior of the rotary regenerator including longitudinal heat conduction in the direction of fluid flow are sufficiently complicated to preclude a complete analytical solution. A few solutions found in the literature restricted to very special cases are discussed.

A numerical finite difference method is presented which will determine the effect of longitudinal heat conduction in rotary regenerators for steady state conditions. In the development no assumptions are made which would restrict the range of parameters for which the analysis would be applicable.

The conduction effect on the regenerator effectiveness was evaluated with the view of obtaining results most useful for the gas turbine regenerator problem, however, these results may also be used for other regenerator problems.

A CDC 1604 digital computer was used to carry out the computations. The results are presented graphically and in tabular form, employing a suitable set of non dimensional parameters. The range of parameters which have been covered are:

$$0.5 \leq C_{min}/C_{max} \leq 1.0$$

$$1.0 \leq C_r/C_{min} \leq 10.0$$

$$1.0 \leq NTUo \leq 20.0$$

$$0.25 \leq (hA)^* \leq 1.0$$

$$0 \leq \lambda \leq 0.2$$

$$0.15 \leq As^* \leq 1.3$$

The author expresses his appreciation to C. P. Howard, Associate Professor, for his direction and encouragement in this work.

TABLE OF CONTENTS

Section	Title	Page
	Abstract	ii
	List of Illustrations and Tables	iv
	Nomenclature	vii
1.	Introduction	1
2.	Method	3
3.	Discussion of Results	18
4.	Summary and Conclusions	20
5.	Bibliography	21
6.	Appendix 1	50
7.	Appendix 2	59

LIST OF ILLUSTRATIONS AND TABLES

Figure Page

1a Illustrative Matrix Arrangement and Fluid Flow 4

1b Schematic Representation of a Rotary Regenerator 6

2 Effectiveness versus Element Area 14

3 $\frac{\Delta\epsilon}{\epsilon}$ versus λ for $C_{min}/C_{max} = 1.0$, $1.0 \leq C_r C_{min} < \infty$, $1.0 \leq NTUo \leq 20.0$ and $0.25 \leq (hA)^* = As^* \leq 1.0$ 46

4 $\frac{\Delta\epsilon}{\epsilon}$ versus λ for $C_{min}/C_{max} = 0.9$, $1.0 \leq C_r/C_{min} < \infty$, $1.0 \leq NTUo \leq 20.0$ and $0.25 \leq (hA)^* = As^* \leq 1.0$ 47

5 $\frac{\Delta\epsilon}{\epsilon}$ versus λ for $C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = 1.0$, $NTUo = 10.0$ 48

6 $\frac{\Delta\epsilon}{\epsilon}$ versus NTUo for $C_{min}/C_{max} = 1.0, 0.9, 0.5$ $C_r/C_{min} = 1.0, 1.5, 2.0, 5.0$, $(hA)^* = As^* = 1.0$, $\lambda = 0.08$ 49

7 Computer Flow Diagram 65

Table

1 Number of Passes to Meet Convergence Criterion 16

2-25 Longitudinal Heat Conduction Effect for:

2 $C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = 1.0$ 22

3 $C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 1.2$, $(hA)^* = As^* = 1.0$ 23

4 $C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 1.4$, $(hA)^* = As^* = 1.0$ 24

5 $C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 2.0$, $(hA)^* = As^* = 1.0$ 25

6 $C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = 1.0$ 26

7 $C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = 0.5$ 27

LIST OF TABLES (CONTINUED)

Table Page

8 $C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 2.0$, $(hA)^* = As^* = 0.5$ 28

9 $C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = 0.5$ 29

10 $C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = 0.25$ 30

11 $C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = 0.25$ 31

12 $C_{min}/C_{max} = 0.9$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = 1.0$ 32

13 $C_{min}/C_{max} = 0.9$, $C_r/C_{min} = 2.0$, $(hA)^* = As^* = 1.0$ 33

14 $C_{min}/C_{max} = 0.9$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = 1.0$ 34

15 $C_{min}/C_{max} = 0.9$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = 0.5$ 35

16 $C_{min}/C_{max} = 0.9$, $C_r/C_{min} = 2.0$, $(hA)^* = As^* = 0.5$ 36

17 $C_{min}/C_{max} = 0.9$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = 0.5$ 37

18 $C_{min}/C_{max} = 0.9$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = 0.25$ 38

19 $C_{min}/C_{max} = 0.9$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = 0.25$ 39

20 $C_{min}/C_{max} = 0.5$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = 1.0$ 40

21 $C_{min}/C_{max} = 0.5$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = 1.0$ 41

22 $C_{min}/C_{max} = 0.5$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = 0.5$ 42

LIST OF TABLES (CONTINUED)

Table Page

23 $C_{min}/C_{max} = 0.5$, $C_r/C_{min} = 10.0$, $(hA)^* = 0.5$ 43

24 $C_{min}/C_{max} = 0.5$, $C_r/C_{min} = 1.0$, $(hA)^* = 0.25$ 44

25 $C_{min}/C_{max} = 0.5$, $C_r/C_{min} = 10.0$, $(hA)^* = 0.25$ 45

NOMENCLATURE

A = heat transfer area on side designated by subscript, sq ft

As_x = solid area available for longitudinal heat conduction on the side of C_{max}, sq ft

As_n = solid area available for longitudinal heat conduction on the side of C_{min}, sq ft

As = total solid area available for longitudinal heat conduction, sq ft

C = heat capacity rate (Wc) of fluid or rotor matrix according to subscript, Btu/hr, deg F

c = specific heat of fluid (constant pressure) or rotor matrix material depending on subscript, Btu/lb, deg F

h = unit conductance for thermal convection heat transfer, Btu/hr, deg F, sq ft

k = unit thermal conductivity, Btu/hr, sq ft, deg F/ft

L = total length of the matrix in the direction of fluid flow, ft

N = number of subdivisions according to subscript

Q = heat transfer rate, Btu/hr

T = temperature of matrix, deg F

$Tx(i,j)$ = subscripted matrix temperature on the side of C_{max}, deg F

$T_{n(f,g)}$ = subscripted matrix temperature on the side of C_{min}, deg F

t = temperature of fluid, deg F

$t_{x(i,j)}$ = subscripted fluid temperature on the side of C_{max}, deg F

$t_{n(f,g)}$ = subscripted fluid temperature on the side of C_{min}, deg F

W = mass flow rate of fluid (lb/hr) or matrix (lb, rev/hr), according to subscript

y = distance of a point from the cold end, measured in units of total length of matrix, $0 \leq y \leq L$

τ = time measured in units of one half period

$A_1, A_2, \ldots\ldots\ldots\ldots\ldots, A_4$
$C_1, C_2, \ldots\ldots\ldots\ldots\ldots, C_4$
$D_1, D_2, \ldots\ldots\ldots\ldots\ldots, D_4$ = constants.
$E_1, E_2, \ldots\ldots\ldots\ldots\ldots, E_4$
$F_1, F_2, \ldots\ldots\ldots\ldots\ldots, F_4$

Subscripts.-

avg = average

c = cold side

h = hot side

i = inlet

min - minimum magnitude

max - maximum magnitude

n - side of C_{min}

o = outlet

r = rotor

x = side of C_{max}

∞ = subscript on e to indicate value extrapolated to an infinite number of elements.

Dimensionless parameters.-

e = exchanger heat transfer effectiveness, ratio of actual to thermodynamically limited maximum possible heat transfer rate

$\frac{\Delta e}{e} = \frac{e_{\lambda=0} - e_{\lambda}}{e_{\lambda=0}}$ = longitudinal heat conduction effect

C_{min}/C_{max} = Capacity rate ratio of fluid flow streams

C_r/C_{min} = capacity rate ratio of rotor matrix to minimum fluid capacity rate

$(hA)^* = (hA)n/(hA)_x$, conductance ratio

NTU = (hA)/C, number of transfer units on side designated by subscript

$NTU_o = NTU_n \left[\frac{\text{[illegible]}}{\text{[illegible]}} \right]$, over-all number of transfer units

$\lambda_x = \frac{k A_x}{\text{[illegible]}}$, conduction parameter on side of C_{max}

$\lambda_n = \frac{k A_n}{C_{min} \text{[illegible]}}$, conduction parameters on side of C_{min}

$\lambda = \frac{k A}{\text{[illegible]} L}$, total conduction parameter

$As^* = As_n/As_x$, conduction area ratic, ratio of solid areas available for heat conduction in the direction of fluid flow

I. Introduction

The regenerative type of heat exchanger has been subjected to extensive treatment within the last thirty years or so and contributions to the theory of regenerators have been made by many writers. The most questionable of the idealizations made in the development of the theory is that of zero matrix conductivity in the direction of fluid flow. The loss due to heat conduction in the matrix material is specially important in the case of high performance regenerators in which a small reduction in the effectiveness will result in a large increase of the heat transfer areas to compensate for the loss. Another important factor is the modern trend toward smaller sizes, since the loss due to conduction of heat in the matrix obviously increases with decreasing length.

Schultz [2][1] solved the problem including conduction for the special case of the balanced regenerator and very large values of C_r/C_{min} (i.e., the capacity rate ratio of rotor matrix to minimum fluid capacity rate). In this case the system of partial differential equations defining the problem for the steady state are reduced to ordinary differential equations and the problem can be solved analytically.

Hahnemann [3] extended the solution to the unbalanced regenerator (i.e., unequal partitions of the cross sections and unequal heat transfer coefficients together with unequal capacity rate ratios of the fluid streams) but still for an infinitely large value of C_r/C_{min}.

London has shown, as reported by Lambertson [1], that a correction factor for conductivity in the direction of fluid flow for approximately

---

[1] Number in brackets refers to bibliography on page 21

equal fluid capacities, of the form:

$$\frac{\Delta \epsilon}{\epsilon} = \frac{k A}{C L}$$

should result in a somewhat pessimistic prediction of the reduction in effectiveness.

It is the objective of this thesis to develop and present a finite difference numerical analysis with solutions to solve for the conduction effect on effectiveness for the rotary regenerator. Since there are six dimensionless parameters required to specify behavior in the most general case, the ranges of the parameters covered in the solutions will be those of most interest to the gas turbine regenerator, however, the analysis will place no restrictions on these parameters.

2. Method

The conventional idealizations and boundary conditions assumed in the derivation of the governing differential equations, including the heat conduction in the direction of fluid flow [4], are the following: (See Fig. 1a)

a. The thermal conductivity of the matrix material is zero in the direction of matrix metal flow. It has a finite value in the direction of fluid flow, and is infinite in the other direction normal to the fluid flow.

b. The specific heats of the two fluids and matrix material are constant with temperature.

c. No leakage of the fluids occurs either due to direct leakage or carry over, and each fluid flow is unmixed.

d. The convective conductance between the fluid and the matrix are constant with flow length.

e. The fluids pass in counterflow directions.

f. Entering fluid temperatures are uniform over the flow inlet cross section and constant with time.

g. The matrix temperature gradient in the direction of fluid flow is zero at the ends.

h. Regular periodic conditions are established for all matrix elements, i.e., steady state condition.

The differential equations derived by Schultz [2] are given in Appendix 1.

It was suggested by Dusinberre and carried out by Lambertson [1] for the case of no conduction, that a rotary regenerator could be divided

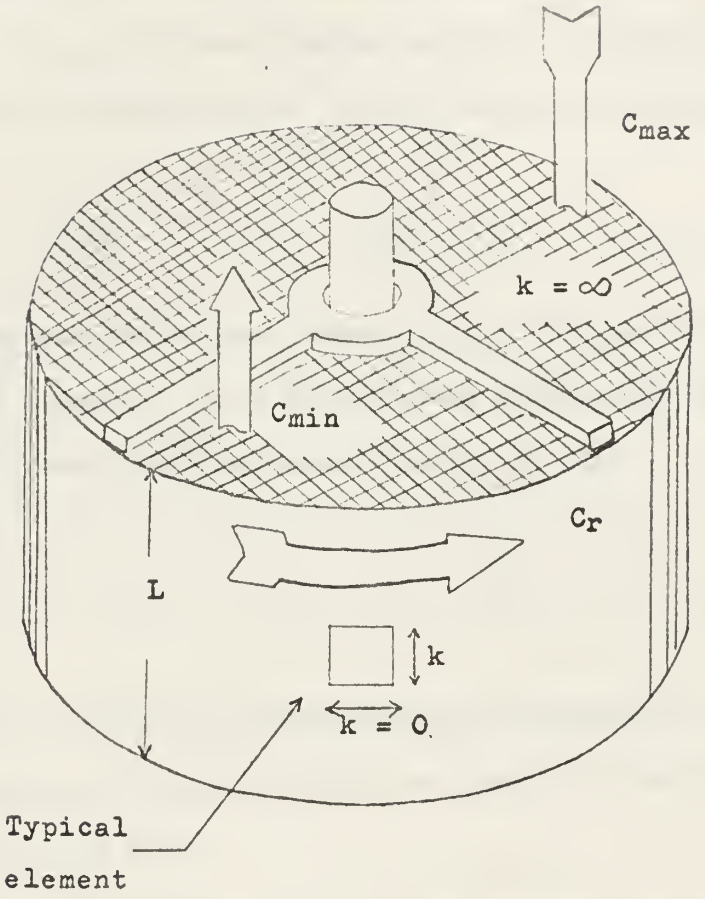


FIG. 1a ILLUSTRATIVE MATRIX ARRANGEMENT AND FLUID FLOW

into finite elements as represented schematically in Fig. 1b. This same method can be extended to the case with conduction. If the elements are considered to be fixed in space, each element can be regarded as a cross flow heat exchanger with a gas stream and a metal stream. For an element on the side of C_{max}, the heat transfer rate by convection is equal to the rate of change in enthalpy of the fluid across the element,

$$\frac{(hA)_x}{N_x N_r} \Delta T_{avg} = \frac{C_{max}}{N_x}\left[t_{x(i,j)} - t_{x(i+1,j)}\right] \tag{1}$$

where ΔT_{avg} represents the mean temperature difference between the fluid and the matrix element. For a small enough element the arithmetic mean temperature difference may be assumed valid, so that

$$\Delta T_{avg} = \frac{1}{2}\left[t_{x(i,j)} + t_{x(i+1,j)}\right] - \frac{1}{2}\left[T_{x(i,j)} + T_{x(i,j+1)}\right]$$

where the fluid and matrix temperatures are subscripted to indicate the average temperatures across the inlet and outlet of the element for simple unmixed cross flow.

Considering an energy balance for the element, the energy transferred to the element by convection plus energy transferred to the element by conduction less the energy transferred out of the element by conduction must equal the energy stored in the element:

$$\frac{C_{max}}{N_x}\left[t_{x(i,j)} - t_{x(i+1,j)}\right] + k\frac{A_x}{N_x}\frac{\left[T_{x(i-1,j)} + T_{x(i-1,j+1)}\right] - \left[T_{x(i,j)} + T_{x(i,j+1)}\right]}{\frac{2L}{N_r}}$$

$$- k\frac{A_x}{N_x}\frac{\left[T_{x(i,j)} + T_{x(i,j+1)}\right] - \left[T_{x(i+1,j)} + T_{x(i+1,j+1)}\right]}{\frac{L}{N_r}} = \frac{C_r}{N_r}\left[T_{x(i,j+1)} - T_{x(i,j)}\right] \tag{2}$$

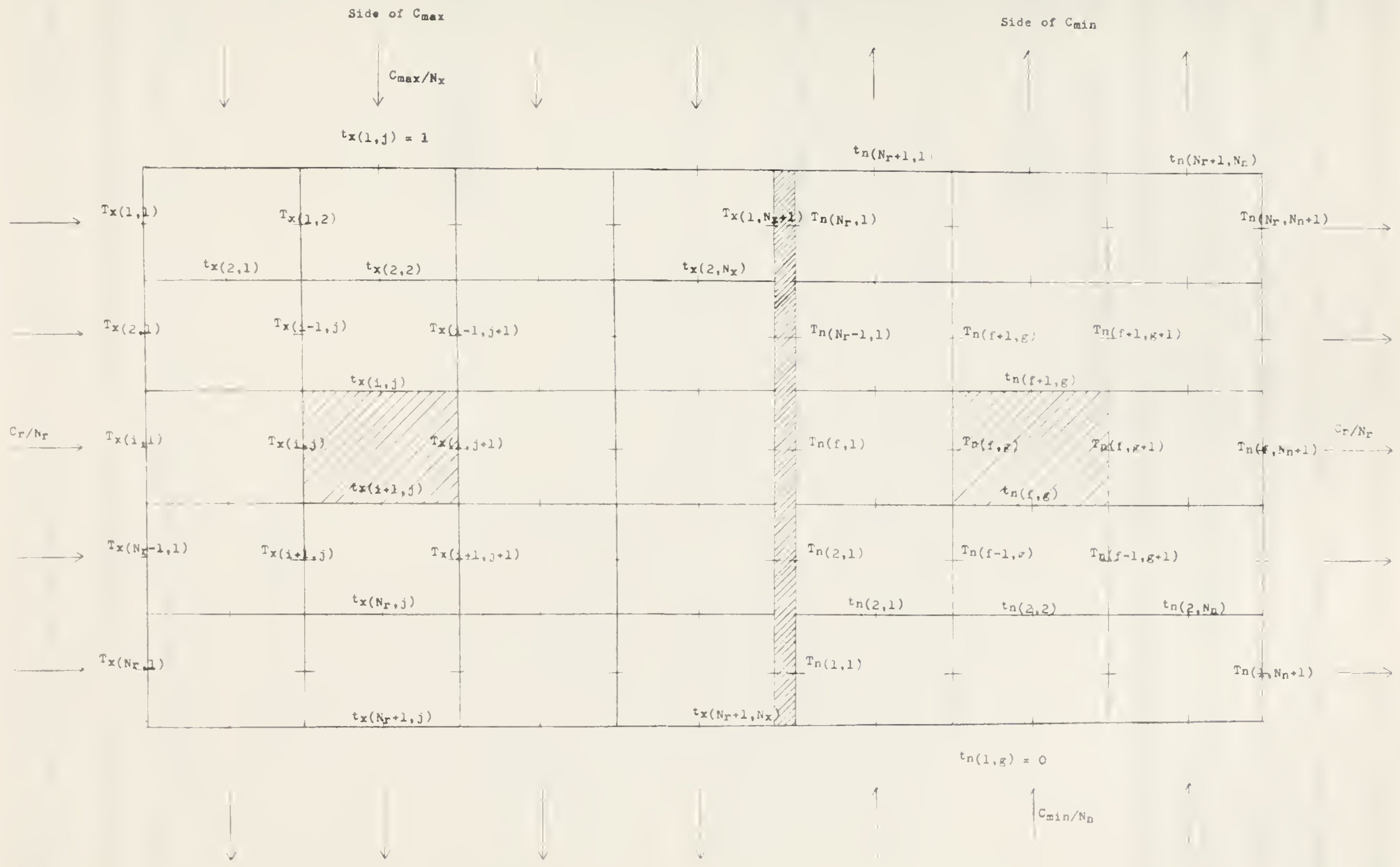


FIG. 1b SCHEMATIC REPRESENTATION OF A ROTARY REGENERATOR

By using equations (1) and (2) the outlet temperatures for an element on the side of C_{max} may be solved for in terms of the remaining temperatures and the result expressed in dimensionless form (Appendix 1).

Since the longitudinal heat conduction is zero at the ends, the energy balance equation is different from equation (2) for the elements of the first and last row. Three different expressions for the matrix outlet temperatures are obtained:

First row elements

$$T_{x\,\text{[illegible]}} = D_1 T_{\text{[illegible]}} - D\,T_{\text{[illegible]}} + D_8 \left[T_{x\,\text{[illegible]}} - T_{x\,\text{[illegible]}\,j+1)} \right] \qquad (3)$$

Middle row elements

$$T_{x\,\text{[illegible]}} = D_1 T_{\text{[illegible]}} - D\,T_{\text{[illegible]}} + D_8 \Big[T_{\text{[illegible]}} + T_{x\,\text{[illegible]}}$$

$$- T_{x(\text{[illegible]}\,j)} + T_{\text{[illegible]}\,j+1)} \Big] \qquad (4)$$

Lsst row elements

$$T_{x\,\text{[illegible]}} = D_1 T_{\text{[illegible]}} - D\,T_{\text{[illegible]}} + D_8 \left[T_{x(\text{[illegible]}} - T_{x\,\text{[illegible]}} \right] \qquad (5)$$

By solving for the fluid outlet temperature on the side of C_{max} in terms of the matrix outlet temperature from equation (1), only one expression is obtained, which holds for all the elements on the side of C_{max},

$$T_{x(i+1,j)} = D_3\, t_{x(i,j)} + D_4 \left[T_{x(i,j)} - T_{x(i,j+1)} \right] \qquad (6)$$

Similarly, for an element on the side of C_{min}

$$\frac{U\,A}{N_h\, N_c}\, \Delta T_{avg} = \frac{C_{min}}{N_c} \left[t_{n(i+1,j)} - T_{n(i,j)} \right] \qquad (7)$$

with

$$\Delta T_{avg} = \frac{1}{2} \left[T_{n(i,j)} + T_{n(i+1,j)} \right] - \frac{1}{2} \left[t_{n(i,j)} + t_{n(i+1,j)} \right] .$$

Energy balance:

$$\frac{C}{N_h} \left[T_{n(i,j)} - T_{\text{[illegible]}} \right] + \lambda\, \frac{A}{N_h}\, \frac{\left[T_{\text{[illegible]}} - T_{\text{[illegible]}} \right] - \left[T_{\text{[illegible]}} - T_{\text{[illegible]}} \right]}{\frac{L}{N_c}}$$

$$- \lambda\, \frac{A}{N_h}\, \frac{\left[T_{(i+1,j)} + T_{\text{[illegible]}} \right] - \left[T_{\text{[illegible]}} - T_{\text{[illegible]}} \right]}{\frac{L}{N_c}} = \frac{C}{N} \left[T_{n(i,j)} - T_{\text{[illegible]}} \right] \qquad (8)$$

By the same method the outlet temperatures of the elements on the side of C_{min} may be obtained using equations (7) and (8).

First row elements

$$T_{n(N,j+1)} = F_6\, t_{n(N,j)} - F_7\, T_{n(N,j)} + F_8 \left[T_{n(N-1,j)} + T_{\text{[illegible]}} \right] \qquad (9)$$

Middle row elements

$$T_{n(i,j+1)} = F_1\, t_{n(i,j)} - F_2\, T_{n(i,j)} + F_3 \left[T_{n(i-1,j)} + T_{\text{[illegible]}} \right.$$

$$\left. + T_{n(i+1,j)} + T_{\text{[illegible]}} \right] \qquad (10)$$

Last row elements

[illegible] (11)

Solving for the fluid outlet temperature for the elements on the side of C_{min} from equation (7) gives

$$t_{[illegible](f,[illegible])} = F_{[illegible]}\, t_{n(f,g)} + F_5 \left[T_{n(f,g)} + T_{n(f,g+1)} \right], \qquad (12)$$

which holds for all the elements on the side of C_{min}. For the values of the constants see Appendix 1.

In addition to the four dimensionless groupings for no conduction,

C_{min}/C_{max} = capacity rate ratio of fluid flow streams

C_r/C_{min} = capacity rate ratio of rotor matrix to minimum fluid capacity rate

$(hA)^*$ = conductance ratio

NTUo = [illegible], over-all number of transfer units;

two additional dimensionless groupings are needed in the coefficients of equations (3), (4), (5), (6), (9), (10), (11), and (12) to take conduction into account. These conduction parameters may be defined as:

$\lambda_{[illegible]} =$ [illegible], conduction parameter on side of C_{max}

$\lambda_{[illegible]} =$ [illegible], conduction parameter on side of C_{min}.

Also [illegible]

since the thermal conductivity and the length of the matrix are the same for both sides, and where

$$\lambda^* = \frac{A_w}{A_x}$$, conduction area ratio.

Another form of defining the conduction parameter is

$$\lambda = \frac{[illegible]}{[illegible]} = \lambda_x \left[1 + \frac{[illegible]}{[illegible]} \right]$$, total conduction parameter

In this case As is the total solid area available for conduction, that is

$$A_s = A_x + A_w .$$

This form offers the advantage that for a given value of the conduction parameter, the resulting effectiveness will not be affected by small changes in As*.

From the schematic representation shown in Fig. 1b, it should be noted that the left edge of the regenerator is physically the same as the right edge and, therefore, the matrix inlet temperature for the elements of the first column on the side of C_{max} is the same as the outlet temperature of the corresponding element of the last column on the side of C_{min}. Expressed mathematically,

$$T_{i(f,1)} = T_{i(f,N_c+1)}$$

where i and f can take the values of 1, 2, 3,...........N_r.
This is referred to as the reversal condition.

From equations (3), (4), (5), (9), (10), and (11) it is seen that in order to solve for the outlet matrix temperature for a particular element, it is necessary to know the matrix temperatures of the next element. An estimate of these values can be made by determining the

temperature distribution for the case of zero conduction in the direction of fluid flow, since the constants D_3 and D_8 in equations (3), (4) and (5) and the constants F_3 and F_8 in equations (9), (10) and (11) are zero in this case.

A temperature scale can be used for which the fluid entrance temperature is zero at the side of C_{min} and unity at the side of C_{max}.

$$t_{n(1,g)} = 0 \qquad \text{for } g = 1,2,3,\ldots\ldots\ldots\ldots N_n$$

$$t_{x(1,j)} = 1 \qquad \text{for } j = 1,2,3,\ldots\ldots\ldots\ldots N_x$$

For calculation purposes a temperature distribution is assumed on the left edge and the problem is solved for the no conduction case in order to obtain an initial estimate of matrix temperatures for working the problem with conduction. This also provides the no conduction effectiveness necessary for comparison.

The outlet temperatures are calculated for every element by repetitive use of equations (3), (4), (5) and (6) depending on the location of the element. The calculation is started with the first element, first column on the side of C_{max} and then working down the column. When the first column of elements is completed, the second column is calculated and so on until the outlet temperatures of all the elements on the side of C_{max} are determined. It was found that convergence could be enhanced if two passes per column were made before proceeding to the next one. The reason for this is that the equations for the outlet temperatures contain the matrix temperatures of the next element which are necessary to estimate.

At the seal represented by the double line in Fig. 1b,

$$T_{x(i,N_x+1)} = T_{n(f,1)}$$

since the matrix outlet temperature for the elements of the last column on the side of C_{max} is physically the same as the inlet temperature of the corresponding element of the first column on the side of C_{min}.

The same method is applied to the side of C_{min} as to the side of C_{max}. If the temperature distribution assumed on the left edge was correct it would then be duplicated on the right (i.e., the reversal condition is fulfilled). If, however, this is not the case, the resulting temperatures on the right side $\left(T_{n(f,N_n+1)}\right)$ are now used on the left $\left(T_{x(i,1)}\right)$ and the procedure is repeated. After each pass (i.e., the complete calculation of a temperature distribution) an energy balance is made; and before the solution is accepted for a particular set of parameters, the heat balance error together with the reversal condition has to be fulfilled to the specified accuracy.

The effectiveness of a heat exchanger is defined (Appendix 2) as

$$e = \frac{C_{max}(t_{xi} - T_{xo})}{C_{min}(t_{xi} - t_{ni})} = \frac{t_{no} - t_{ni}}{t_{xi} - t_{ni}} .$$

From the conditions of the problem the above expression for effectiveness reduces to

$$e = \left[t_{n(N+1,j)}\right]_{avg} .$$

The heat balance error was computed from

$$\text{error} = 1 - \frac{\frac{C_{max}}{C_{min}}\left[T_{n(N_r+1,j)}\right]_{avg}}{1 - \left[T_{x(N+1,j)}\right]_{avg}} .$$

That the solution will converge (i.e., the reversal condition met and the heat balance error converge to zero) will depend on the number of elements used. More important than this is the relation existing between the number of subdivisions for the three streams. The sufficient but not necessary conditions for convergence are (Appendix 1) that

$$N_x > \left(\frac{\lambda_x}{A^*}\right) \frac{1}{C_{min}} N_r^2$$

$$N_n > \lambda_n \frac{1}{C_r/C_{min}} N_r^2$$

Fig. 2 is a plot of the different values of effectiveness obtained for the following set of parameters·

$C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 2.0$, $(hA)^* = 1.0$, $NTUo = 10.0$ $\lambda = 0.1$, $As^* = 1.0$.

Several curves are obtained depending on the relation existing between the number of subdivisions for every stream. To extrapolate the effectiveness to an infinite number of elements, these curves may be approximated by straight lines. The error introduced will depend on the number of elements used and will not be constant for all the range of the parameters. As an illustration consider the set of parameters used to plot Fig. 2. Three cases are investigated:

1) $N_x = N_n = N_r/2$

2) $N_x = N_n = N_r$

3) $N_x = N_n = 2N_r$

A linear extrapolation is performed using the values of effectiveness obtained for values of $\dfrac{1}{N_r(N_x + N_n)}$. From the results it is found that the minimum N_r necessary to give an error in the extropolated effectiveness of less than 0.01% is 18, for which $e_\infty = 76.57\%$

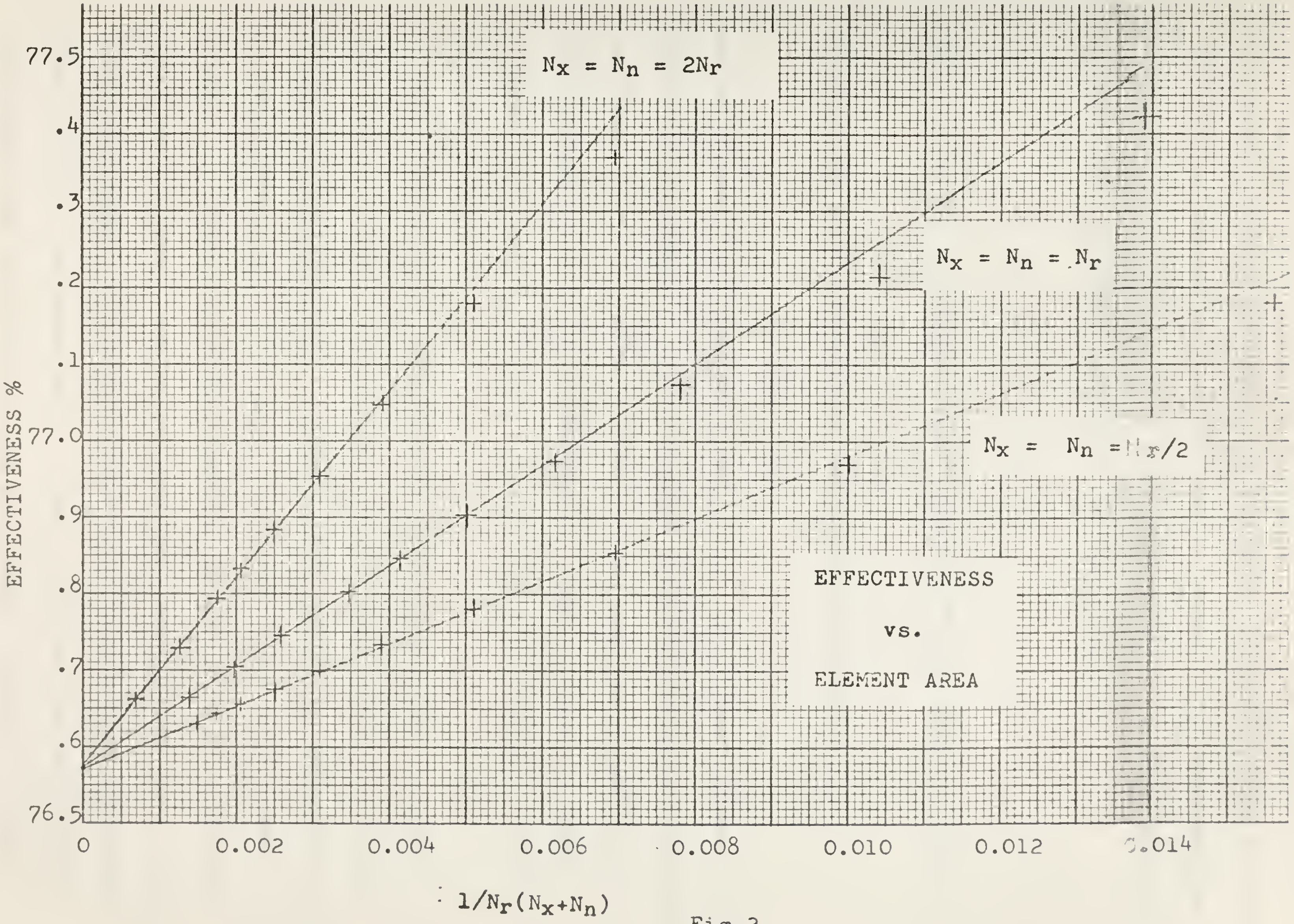

EFFECTIVENESS vs. ELEMENT AREA
Nx = Nn = 2Nr
Nx = Nn = Nr
Nx = Nn = Nr/2
EFFECTIVENESS %
77.5
.4
.3
.2
.1
77.0
.9
.8
.7
.6
76.5
0
0.002
0.004
0.006
0.008
0.010
0.012
0.014
1/Nr(Nx+Nn)

Fig 2

for the three cases. The determining factor in the extrapolation is the number of matrix elements. The higher the value of N_r used for the extrapolation the smaller the error in the straight line approximation. Unfortunately, an increase in N_r will bring a higher increase of N_x and N_n, since the convergence condition for this example is

$$N_x = N_n > 0.05\, N_r^2 .$$

That this is only a sufficient condition can be seen from the case of

$$N_x = N_n = N_r/2$$

for which the solution converges for values of $N_r > 5$; and when

$$N_x = N_n = N_r$$

the solution will still converge for $N_r > 20$. It is true, however, that the closer the number of matrix elements are to the sufficient convergence condition, the fewer passes it takes for the solution to converge to the specified error in the effectiveness. Table 1 shows the number of passes necessary to obtain a solution with a heat balance error of less than 0.005%.

For a given set of parameters the effectiveness was determined first using one set of subdivisions and then recalculated increasing the number of subdivisions. A linear extrapolation was performed to obtain the effectiveness corresponding to an infinite number of elements. The accuracy for each value of effectiveness was specified to be of four significant figures. When these values were extrapolated to an infinite number of elements, the accuracy was reduced to only three significant figures since a compromise had to be made with the time available for computations and the number of runs to be made. However, the method

TABLE 1

Number of passes necessary to obtain an effectiveness with a heat balance error of less than 0.005% for the following set of parameters:

$C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 2.0$, $(hA)^* = 1.0$,

$NTUo = 10.0$, $\lambda = 0.1$, $As^* = 1.0$

N_r	N_x	N_n	No. of passes	e_∞^* %
12	12	12	27	76.581
12	24	24	22	76.581
14	7	7	41	76.578
14	14	14	34	76.576
14	28	28	27	76.576
16	16	16	41	76.574
16	32	32	32	76.574
18	9	9	62	76.573
18	18	18	50	76.572
18	36	36	38	76.572
20	20	20	60	76.572
22	22	22	71	76.571

* e_∞ is obtained by a linear extrapolation of the effectiveness for N_r and N_r-1.

imposes no restriction as to the ultimate accuracy of the solution.

When working with very high values of C_r/C_{min} the computation time becomes considerably long. This is due to the fact that the final temperatures differ very little from the assumed initial temperatures and a large number of elements is required. The starting estimates of matrix temperatures then become very important. For the limiting case of C_r/C_{min} equal to infinite, there is no temperature variation in the direction of metal flow so that the method without modifications is not applicable. For this case, however, the exact solution of Hahnemann [3] is available.

The Fortran system was used to program the above method for the CDC 1604 digital computer. The matrix notation employed in the equations derived previously is very convenient in writing up the program, since several loops can be made by just changing the subscripts of the variables involved. The complete computer program is given in Appendix 2.

3. Discussion of results

The solutions generated are presented in tabular form in Tables 2 through 25 and graphically in Figs. 3 and 4. The accuracy in the effectiveness was specified to be of three significant figures. It was found that for

$$0.7\ (hA)^{*} \leq As^{*} \leq 1.3\ (hA)^{*}$$

the resulting values of effectiveness were almost identical to the ones calculated for

$$As^{*} = (hA)^{*}$$

so that these were the values used in preparing the Tables.

By using NTUo and λ as defined in the nomenclature, which contain $(hA)^{*}$ and As^{*} explicitly, the influence of these two parameters on the conduction effect, $\frac{\Delta e}{e}$, were found to be very small for values of $C_{min}/C_{max} \geq 0.9$ and $\lambda \leq 0.1$, which is the range for the gas turbine regenerator problem.

Figs. 3 and 4 show the effect of conduction on effectiveness for values of $C_{min}/C_{max} = 1.0$ and 0.9 respectively, with different values of NTUo, C_r/C_{min} and λ, up to a limit of 10%. The conduction effect continues to increase with increasing λ. For small values of λ this effect increases nearly exponentially. By further increasing λ, a saturation is reached which approaches the limiting condition of $\lambda = \infty$. Fig. 5 is a representative curve which shows the limiting influence on the conduction effect by continuously increasing λ for a given set of parameters.

The conduction effect for values of $C_r/C_{min} = 10.0$ are almost identical with the values reported by Schultz [2] and Hahnemann [3], [6] for the limiting case of $C_r/C_{min} = \infty$.

For small values of NTUo the effect of increasing C_r/C_{min} is to increase slightly the conduction effect, as can be seen from Fig. 6. For higher values of NTUo the effect of conduction is reduced with increasing C_r/C_{min}.

The effect of increasing NTUo upon $\frac{\Delta e}{e}$ is intimately related to the value of C_r/C_{min} and C_{min}/C_{max}. This effect can best be explained by observing Fig. 6. For the case where $C_{min}/C_{max} = 1.0$ the effect of increasing NTUo will result in an increase in $\frac{\Delta e}{e}$ regardless of the value of C_r/C_{min}; of course, the higher the value of C_r/C_{min} the smaller this effect will be for relatively high values of NTUo. Now, by reducing C_{min}/C_{max} and increasing C_r/C_{min} a point is reached in which an increase in the NTUo will result in a reduction in $\frac{\Delta e}{e}$. The point where this occurs will depend upon the value of the conduction parameter λ.

4. Summary and Conclusions

A finite difference numerical analysis is presented which will determine the effect of the longitudinal heat conduction in rotary regenerators for steady state conditions.

The ranges of governing parameters which have been covered are:

$0.5 \leq C_{min}/C_{max} \leq 1.0$

$1.0 \leq C_r/C_{min} \leq 10.0$

$1.0 \leq NTUo \leq 20.0$

$0.25 \leq (hA)^* \leq 1.0$

$0 \leq \lambda \leq 0.2$

$0.15 \leq As^* \leq 1.3$

The ranges of parameters of interest for gas turbine regenerator application are plotted in Figs. 3 and 4. A more accurate determination of the conduction effect than that given in the figures can be made by interpolating values from the Tables.

The computer program used to carry out the calculations on the CDC 1604 digital computer is included in Appendix 2. This program imposes no restrictions on the parameters for which the analysis would be applicable. It is recommended, however, that for values of $C_r/C_{min} > 10$ the solution for $C_r/C_{min} = 10$ be used. For this case the computer time for a solution increases considerably and the error introduced by assuming $C_r/C_{min} = 10$ is negligible.

BIBLIOGRAPHY

1. "Performance Factors of a Periodic Flow Heat Exchanger", by Tomme J. Lambertson, Tran A.S.M.E., vol 80, April 1958, pp 586-592.

2. "Regenerators with Longitudinal Heat Conduction", by B. H. Schultz, IME-ASME General Discussion on Heat Transfer, London, England, 1951, pp 440-443.

3. "Approximate Calculation of Thermal Ratios in Heat Exchangers Including Heat Conduction in Direction of Flow," by H. W. Hahnemann, National Gas Turbine Establishment Memorandum No. M36, 1948.

4. "Accomplished Calculations of Heat Exchange in Regenerators", by H. Hausen, MAP Reports and Translations No. 312, November 1, 1946.

5. "The Periodic Flow Regenerator", by J. E. Coppage and A. L. London, Trans. ASME, vol 75, July 1953, pp 779-787.

6. "A Design Manual for Regenerative Heat Exchangers of the Rotary Type", by Harold H. Sogin and Kamal-Eldin Hassan, WADC Technical Report 55-13.

TABLE 2 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = 1.0$

	e %	Conduction effect, $\frac{\Delta\epsilon}{\epsilon}$ %							
NTUo \ λ	0	0.01	0.02	0.04	0.08	0.14	0.16	0.20	0.32
1	46.64	0.42	0.78	1.48	2.61	3.94	4.30	4.95	6.42
2	60.06	0.58	1.11	2.10	3.81	5.93	6.54	7.65	10.03
3	66.72	0.68	1.32	2.50	4.58	7.17	7.92	9.30	12.60
4	70.86	0.77	1.48	2.82	5.17	8.08	8.93	10.48	14.22
5	73.75	0.84	1.63	3.09	5.65	8.81	9.73	11.41	15.44
6	75.92		1.76		6.05	9.42		12.16	
7	77.63		1.87		6.41	9.93		12.80	
8	79.01		1.97		6.72	10.38		13.34	
9	80.16		2.07		7.01	10.78		13.82	
10	81.15	1.12	2.16	4.05	7.26	11.13	12.23	14.24	18.97
12	82.74		2.32		7.71	11.74		14.95	
14	84.00		2.46		8.10	12.25		15.54	
16	85.01		2.60		8.44	12.69		16.04	
18	85.86		2.72		8.74	13.08		16.47	
20	86.58	1.49	2.83	5.17	9.01	13.42	14.60	16.85	21.96
40	90.37	1.92	3.55	6.30	10.54		16.58		24.25
100	93.87	2.70	4.78	7.97	12.55		18.79		26.54
500	97.87	4.04	6.80	10.57	15.42		21.65		29.27
1000	98.81	4.39	7.30	11.17	16.07		22.30		29.88

TABLE 3 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 1.2$, $(hA)^* = As^* = 1.0$

	e %	Conduction effect, $\frac{\Delta e}{e}$ %			
NTUo \ λ	0	0.02	0.08	0.14	0.20
1	47.62	0.81	2.66	4.01	5.04
2	61.97	1.13	3.88	6.05	7.81
3	69.15	1.33	4.64	7.28	9.46
4	73.61	1.49	5.21	8.18	10.63
5	76.72	1.62	5.66	8.89	11.54
6	79.04	1.73	6.05	9.47	12.28
7	80.85	1.84	6.38	9.96	12.89
8	82.31	1.93	6.67	10.38	13.40
9	83.53	2.01	6.93	10.76	13.86
10	84.55	2.09	7.16	11.08	14.26
11	85.44	2.16	7.37	11.38	14.61
12	86.21	2.22	7.56	11.65	14.93
13	86.89	2.28	7.74	11.89	15.22
14	87.49	2.34	7.90	12.11	15.48
15	88.03	2.40	8.05	12.31	15.71
16	88.52	2.45	8.19	12.50	15.93
17	88.96	2.50	8.32	12.67	16.13
18	89.37	2.54	8.44	12.83	16.31
19	89.74	2.59	8.56	13.00	16.48
20	90.09	2.63	8.66	13.12	16.64

TABLE 4 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 1.4$, $(hA)^* = As^* = 1.0$

	ε %	Conduction effect, $\frac{\Delta\varepsilon}{\varepsilon}$ %							
NTUo \ λ		.01	.02	.04	.08	.14	.16	.2	.32
1	48.23		.82		2.69	4.05		5.10	
2	63.16		1.14		3.93	6.12		7.90	
3	70.65		1.33		4.67	7.33		9.53	
4	75.27		1.48		5.20	8.20		10.67	
5	78.45		1.60		5.63	8.86		11.54	
6	80.82		1.70		5.98	9.40		12.23	
7	82.65		1.79		6.28	9.85		12.80	
8	84.12		1.87		6.54	10.24		13.28	
9	85.33		1.94		6.76	10.57		13.69	
10	86.34		2.00		6.96	10.86		14.05	
12	87.96		2.12		7.31	11.36		14.64	
14	89.20		2.22		7.59	11.75		15.12	
16	90.18		2.3		7.83	12.08		15.51	
18	90.98		2.37		8.04	12.36		15.84	
20	91.65		2.44		8.22	12.61		16.12	

TABLE 5 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 2.0$, $(hA)^* = As^* = 1.0$

	e %	Conduction effect, $\frac{\Delta e}{e}$ %							
NTUo \ λ	0	.01	.02	.04	.08	.14	.16	.2	.32
1	49.12		.84		2.73	4.11		5.17	
2	64.91		1.16		4.0	6.22		8.04	
3	72.80		1.34		4.70	7.40		9.37	
4	77.60		1.47		5.18	8.19		10.70	
5	80.86		1.56		5.54	8.78		11.47	
6	83.22		1.64		5.83	9.23		12.07	
7	85.03		1.71		6.07	9.61		12.55	
8	86.46		1.76		6.27	9.92		12.95	
9	87.63		1.82		6.44	10.18		13.28	
10	88.59		1.86		6.60	10.41		13.57	
12	90.11		1.94		6.85	10.79		14.04	
14	91.25		2.01		7.05	11.08		14.41	
16	92.14		2.06		7.22	11.33		14.71	
18	92.86		2.11		7.36	11.53		14.95	
20	93.45		2.15		7.49	11.71		15.16	
500	99.65	1.40	2.72	5.11	9.10		15.18		23.40
1000	99.80	1.40	2.72	5.13	9.14		15.24		23.47

TABLE 6 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = 1.0$

	e %	Conduction effect, [illegible] %			
NTUo	0	.02	.08	.14	.2
1	49.96	.85	2.77	4.16	5.24
2	66.59	1.18	4.04	6.29	8.12
3	74.91	1.35	4.72	7.41	9.63
4	79.89	1.46	5.13	8.10	10.57
5	83.22	1.53	5.42	8.57	11.21
6	85.59	1.58	5.62	8.92	11.67
7	87.38	1.62	5.78	9.19	12.03
8	88.76	1.65	5.91	9.40	12.31
9	89.87	1.68	6.02	9.57	12.54
10	90.78	1.70	6.10	9.71	12.73
12	92.18	1.73	6.24	9.94	13.03
14	93.20	1.76	6.34	10.11	13.26
16	93.99	1.76	6.43	10.25	13.45
18	94.61	1.80	6.49	10.36	13.59
20	95.11	1.81	6.55	10.45	13.71

TABLE 7 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = 0.5$

	ε %	Conduction effect, $\frac{\Delta\varepsilon}{\varepsilon}$ %					
NTUo \ λ	0	0.015	0.03	0.06	0.12	0.24	0.48
1	46.62	.62	1.17	2.12	3.64	5.72	8.05
2	60.08	.85	1.64	3.04	5.38	8.82	13.03
3	66.75	1.01	1.95	3.63	6.46	10.67	15.90
4	70.90	1.14	2.19	4.09	7.26	11.98	17.83
5	73.79	1.25	2.40	4.47	7.91	12.98	19.26
10	81.19	1.66	3.15	5.77	9.98	15.98	23.2
20	86.62	2.19	4.07	7.22	12.05	18.65	26.36
40	90.46	2.69	4.92	8.51	13.77	20.73	28.69
100	93.61	3.24	5.84	9.85	15.44	22.60	30.64
500	95.57	3.51	6.36	10.66	16.48	23.76	31.91
1000	95.83	3.55	6.43	10.77	16.62	23.92	32.10

TABLE 8 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 2.0$, $(hA)^* = As^* = .5$

	e %	Conduction effect, [illegible] %						
NTUo \ λ	0	.015	.03	.06	.12	.21	.24	.3
1	49.11		1.22		3.79	5.51		6.75
2	64.92		1.70		5.62	8.47		10.64
3	72.81		1.97		6.66	10.10		12.78
4	77.61		2.16		7.32	11.17		14.17
5	80.86		2.30		7.82	11.95		15.16
6	83.23		2.42		8.22	12.55		15.92
7	85.04		2.51		8.54	13.03		16.52
8	86.47		2.60		8.81	13.43		17.01
9	87.63		2.67		9.04	13.76		17.42
10	88.59		2.73		9.24	14.05		17.76
12	90.11		2.84		9.57	14.52		18.32
14	91.25		2.94		9.83	14.88		18.76
16	92.14		3.01		10.05	15.18		19.11
18	92.86		3.08		10.23	15.43		19.40
20	93.45		3.14		10.38	15.63		19.64
500	99.60	2.12	3.95	7.15	12.29		19.63	28.54
1000	99.79	2.17	4.0	7.21	12.37		19.73	28.67

TABLE 9 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = .5$

	e %	Conduction effect, $\frac{\Delta e}{e}$ %					
NTUo \ λ	0	.015	.03	.06	.12	.24	.48
1	49.96	0.65	1.23	2.24	3.84	6.04	8.51
2	66.59	0.90	1.72	3.20	5.67	9.32	13.84
3	74.91	1.02	1.97	3.70	6.62	11.04	16.64
4	79.89	1.10	2.12	4.00	7.21	12.09	18.34
5	83.22	1.15	2.23	4.21	7.61	12.80	19.48
10	90.78	1.28	2.49	4.73	8.59	14.51	22.13
20	95.11	1.36	2.65	5.05	9.20	15.58	23.76

TABLE 10 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = .25$

	e %	Conduction effect, $\frac{\Delta e}{e}$ %					
NTUo \ λ	0	.025	.05	.1	.2	.4	.8
1	46.59	1.02	1.90	3.38	5.57	8.25	10.84
2	60.13	1.43	2.69	4.88	8.22	12.50	16.84
3	66.84	1.70	3.21	5.80	9.78	14.88	20.07
4	70.10	1.92	3.61	6.50	10.88	16.47	22.16
5	73.90	2.10	3.94	7.05	11.73	17.65	23.66
10	81.28	2.74	5.05	8.81	14.25	20.93	27.63
20	86.64	3.52	6.27	10.56	16.50	23.61	30.62

TABLE 11 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = 1.0$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = .25$

	e %	Conduction effect, $\frac{\Delta e}{}$ %					
NTUo λ	0	.025	.05	.1	.2	.4	.8
1	49.96	1.07	1.99	7.92	5.84	8.68	11.49
2	66.60	1.47	2.79	5.06	8.59	13.21	18.05
3	74.91	1.68	3.20	5.84	10.00	15.53	21.43
4	79.89	1.80	3.44	6.32	10.86	16.93	23.48
5	83.22	1.88	3.61	6.64	11.44	17.88	24.86
10	90.78	2.1	4.03	7.44	12.83	20.10	28.02
20	95.11	2.23	4.29	7.92	13.69	21.46	29.83

TABLE 12 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = .9$, $C_r/C_{min} = 1.0$, $(hA)^* = A_s^* = 1.0$

	ε %	Conduction effect, $\frac{\Delta\varepsilon}{\varepsilon}$ %					
NTUo	0	.01	.02	.04	.08	.16	.32
1	47.63	0.38	0.73	1.36	2.41	3.98	5.95
2	61.56	0.53	1.02	1.93	3.53	6.08	9.62
3	68.46	0.62	1.21	2.30	4.24	7.37	11.60
4	72.74	0.70	1.36	2.60	4.78	8.32	13.33
5	75.72	0.76	1.49	2.84	5.22	9.06	14.50
10	83.29	1.01	1.96	3.70	6.70	11.30	17.95
20	88.69	1.32	2.52	4.66	8.21	13.56	20.[illegible]
40	92.41	1.68	3.15	5.64	9.58	15.32	22.74
100	95.50	2.27	4.08	6.94	11.17	17.10	24.67

TABLE 13 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = .9$, $C_r/C_{min} = 2.0$, $(hA)^* = As^* = 1.0$

	ε %	Conduction effect, $\frac{\Delta\varepsilon}{\varepsilon}$ %					
NTUo	0	.01	.02	.04	.08	.16	.32
1	50.30	0.40	0.77	1.43	2.53	4.17	6.23
2	66.92	0.55	1.07	2.03	3.71	6.41	10.16
3	75.30	0.63	1.24	2.37	4.37	7.66	12.36
4	80.40	0.69	1.35	2.59	4.82	8.50	13.80
5	83.85	0.73	1.43	2.76	5.14	9.10	14.82
10	91.99	0.85	1.67	3.23	6.04	10.71	17.44
20	96.82	0.90	1.79	3.49	6.59	11.74	19.11

TABLE 14 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = .9$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = 1.0$

	e %	Conduction effect, $\frac{\Delta e}{e}$ %					
NTUo λ	0	.01	.02	.04	.08	.16	.32
1	51.22	0.41	0.78	1.45	2.56	4.23	6.32
2	68.81	0.56	1.09	2.06	3.77	6.49	10.28
3	77.67	0.63	1.24	2.38	4.39	7.68	12.38
4	82.98	0.68	1.33	2.56	4.76	8.39	13.65
5	86.52	0.70	1.39	2.68	5.00	8.87	14.49
10	94.36	0.75	1.48	2.88	5.48	9.87	16.35
20	98.36	0.65	1.33	2.70	5.35	10.03	17.08

TABLE 15 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = .9$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = .5$

	e %	Conduction effect, $\frac{\Delta e}{e}$ %					
NTUo	0	.015	.03	.06	.12	.24	.48
1	47.53	0.54	1.03	1.88	3.23	5.11	7.23
2	61.40	0.74	1.43	2.67	4.76	7.89	11.81
3	68.26	0.87	1.69	3.17	5.70	9.55	14.48
4	72.51	0.97	1.88	3.55	6.40	10.73	16.29
5	75.47	1.06	2.05	3.86	6.94	11.63	17.64
10	82.98	1.35	2.61	4.89	8.69	14.33	21.38
20	88.34	1.68	3.23	5.97	10.39	16.70	24.35

TABLE 16 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = .9$, $C_r/C_{min} = 2.0$, $(hA)^* = As^* = .5$

	e %	Conduction effect, $\frac{\Delta e}{e}$ %			
NTUo \ λ	0	.03	.12	.21	.30
1	50.27	1.03	3.40	4.96	6.09
2	66.88	1.53	5.08	7.71	9.73
3	75.23	1.77	6.01	9.23	11.76
4	80.32	1.93	6.63	10.24	13.08
5	83.77	2.05	7.09	10.97	14.03
6	86.28	2.14	7.45	11.53	14.75
7	88.18	2.22	7.74	11.98	15.32
8	89.69	2.29	7.97	12.34	15.78
9	90.90	2.34	8.17	12.64	16.17
10	91.91	2.38	8.34	12.90	16.49
12	93.47	2.45	8.60	13.31	17.00
14	94.62	2.50	8.80	13.61	17.39
16	95.50	2.54	8.96	13.85	17.67
18	96.20	2.56	9.07	14.03	17.90
20	96.76	2.57	9.16	14.18	18.09

TABLE 17 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = .9$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = .5$

	e %	Conduction effect, [illegible] %					
NTUo	0	0.015	0.03	0.06	0.12	0.24	0.48
1	51.22	0.58	1.10	2.01	3.45	5.45	7.73
2	68.80	0.80	1.55	2.89	5.15	8.55	12.83
3	77.66	0.92	1.78	3.35	6.04	10.19	15.56
4	82.98	0.98	1.91	3.62	6.58	11.18	17.22
5	86.51	1.02	1.99	3.79	6.39	11.85	18.33
10	94.36	1.07	2.12	4.11	7.65	13.29	20.77
20	98.35	0.96	1.96	3.97	7.71	13.80	21.91

TABLE 18 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = .9$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = .25$

	e %	Conduction effect, $\frac{\Delta e}{e}$ %					
NTUo	0	0.025	0.05	0.1	0.2	0.4	0.8
1	47.40	0.87	1.62	2.90	4.82	7.21	9.57
2	61.29	1.18	2.26	4.13	7.08	10.98	15.09
3	68.13	1.39	2.65	4.87	8.39	13.10	18.12
4	72.37	1.54	2.94	5.41	9.32	14.54	20.11
5	75.31	1.66	3.17	5.83	10.03	15.61	21.54
10	82.74	2.05	3.92	7.17	12.16	18.61	25.35
20	88.02	2.43	4.67	8.45	14.04	21.04	28.16

TABLE 19 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = .9$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = .25$

	e %	Conduction effect, $\frac{\Delta\epsilon}{\epsilon}$ %					
NTU \ λ	0	0.02	0.05	0.1	0.2	0.4	0.8
1	51.22	0.93	1.74	3.10	5.16	7.74	10.36
2	68.80	1.29	2.45	4.49	7.72	12.06	16.74
3	77.66	1.47	2.82	5.21	9.06	14.32	20.12
4	82.98	1.58	3.03	5.64	9.37	15.70	22.10
5	86.51	1.64	3.17	5.93	10.41	16.62	23.57
10	94.36	1.74	3.41	6.48	11.56	18.67	26.62
20	98.35	1.59	3.26	6.46	11.69	19.54	28.00

TABLE 20 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = 0.5$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = 1.0$

	e %	Conduction effect, $\frac{\Delta e}{e}$ %					
NTUo \ λ	0	0.01	0.02	0.04	0.08	0.16	0.32
2	66.89	0.35	0.68	1.29	2.37	4.15	6.71
3	74.01	0.41	0.80	1.53	2.84	5.02	8.25
4	78.16	0.45	0.89	1.71	3.19	5.65	9.29
5	80.89	0.49	0.97	1.86	3.46	6.12	10.06
10	87.15	0.65	1.26	2.39	4.38	7.57	12.15
20	91.07	0.86	1.64	3.04	5.37	8.94	13.87

TABLE 21 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = 0.5$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = 1.0$

	e %	Conduction effect, $\frac{\Delta e}{e}$ %					
NTUo λ	0	0.01	0.02	0.04	0.08	0.16	0.32
1	56.42	0.28	0.53	0.98	1.74	2.90	4.37
2	77.34	0.37	0.71	1.35	2.49	4.36	7.06
3	87.30	0.38	0.74	1.43	2.68	4.82	8.03
4	92.60	0.35	0.70	1.36	2.60	4.80	8.29
5	95.59	0.31	0.62	1.22	2.39	4.53	8.1
10	99.62	0.10	0.22	.49	1.14	2.68	5.96
20	99.99	0.01	0.01	0.05	0.23	1.02	3.65

TABLE [illegible] LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = 0.5$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = 0.5$

	ε %	Conduction effect, $\frac{\Delta\varepsilon}{\varepsilon}$ %					
NTU₀ \ λ	0	0.01	0.02	0.04	0.08	0.16	0.32
1	50.97	0.28	.53	0.98	1.71	2.75	3.99
2	65.65	0.35	.69	1.32	2.40	4.15	6.56
3	72.44	0.39	.78	1.49	2.78	4.92	8.04
4	76.43	0.42	.87	1.62	3.04	5.46	9.04
5	79.10	0.45	0.89	1.72	3.35	5.86	9.79
10	85.44	0.56	1.09	2.12	4.00	7.[illegible]8	11.95
20	89.72	0.74	1.44	2.72	4.95	8.58	13.90

TABLE [illegible] LONGITUDINAL HEAT CONDUCTION EFFECT

C_{min}/C_{max} = [illegible] , [illegible]$_{min}$ = 10.0 , $(hA)^{*}$ = [illegible] = 0.5

	ε %	Conduction effect, $\frac{\Delta\varepsilon}{\varepsilon}$ %					
NTU \ λ	0	0.01	0.02	0.04	0.08	0.16	0.32
1	56.41	[illegible]	0.61	1.01	1.95	3.14	4.54
2	77.32	0.43	0.83	1.57	2.87	4.94	7.76
3	87.27	0.44	0.87	1.66	3.14	5.61	9.12
4	92.57	0.42	0.82	1.62	3.10	5.71	9.73
5	95.56	0.37	0.74	1.47	2.90	5.53	9.76
10	99.61	0.13	0.22	0.63	1.54	3.74	8.13
20	99.99	0.01	0.02	0.08	0.43	1.92	6.06

TABLE 24 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = 0.5$, $C_r/C_{min} = 1.0$, $(hA)^* = As^* = 0.25$

	ϵ %	Conduction effect, $\frac{\Delta\epsilon}{\epsilon}$ %					
NTUo \ λ	0	.01	.02	.04	.08	.16	.32
1	50.21	0.34	0.65	1.18	2.01	3.12	4.04
2	64.39	0.38	0.75	1.42	2.61	4.47	6.89
3	70.98	0.39	0.77	1.51	2.89	5.19	8.38
4	74.91	0.39	0.79	1.58	3.09	5.70	9.43
5	77.57	0.40	0.81	1.64	3.26	6.11	10.24
10	84.12	0.49	0.99	1.99	3.97	7.53	12.71
20	88.72	0.66	1.30	2.54	4.92	9.07	14.97

TABLE 25 LONGITUDINAL HEAT CONDUCTION EFFECT

$C_{min}/C_{max} = 0.5$, $C_r/C_{min} = 10.0$, $(hA)^* = As^* = 0.25$

	e %	Conduction effect, [illegible] %					
NTUo	0	.01	.02	.04	.08	.16	.32
1	56.40	0.43	0.81	1.47	2.51	3.92	5.46
2	77.30	0.58	1.11	2.10	3.79	6.36	9.59
3	87.24	0.60	1.18	2.27	4.24	7.42	11.66
4	92.53	0.57	1.12	2.22	4.28	7.77	12.62
5	95.53	0.50	1.02	2.05	4.10	7.73	12.99
10	99.60	0.18	0.41	1.01	2.59	6.18	12.19
20	99.99	0.01	0.04	0.22	1.13	4.29	10.59

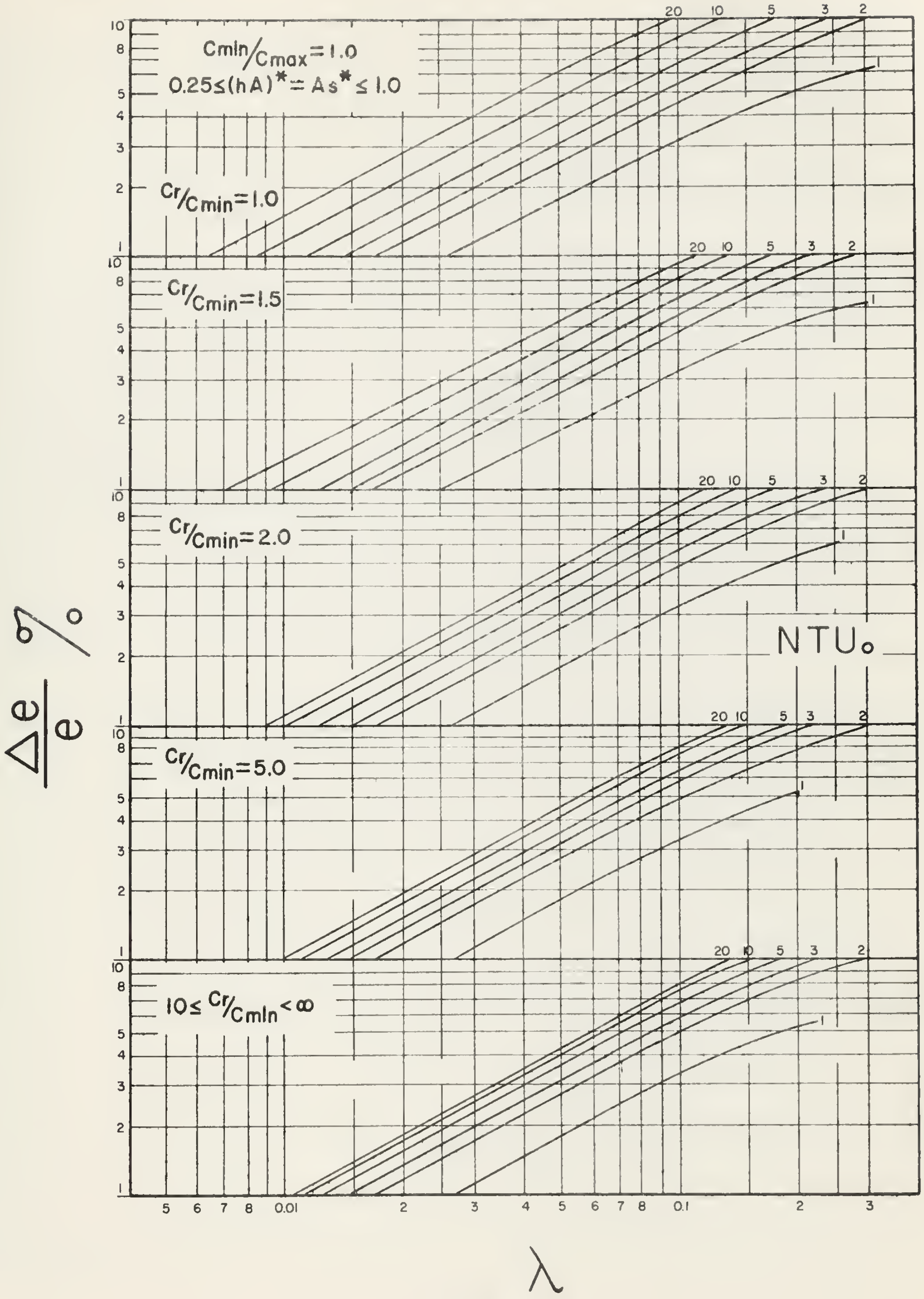

$\frac{\Delta e}{e}$ %
λ
NTU₀
$C_{min}/C_{max} = 1.0$
$0.25 \leq (hA)^* = A_s^* \leq 1.0$
$C_r/C_{min} = 1.0$
$C_r/C_{min} = 1.5$
$C_r/C_{min} = 2.0$
$C_r/C_{min} = 5.0$
$10 \leq C_r/C_{min} < \infty$
20
10
5
3
2
1
8
4
5
6
7
8
0.01
2
3
4
5
6
7
8
0.1
2
3

FIG. 3

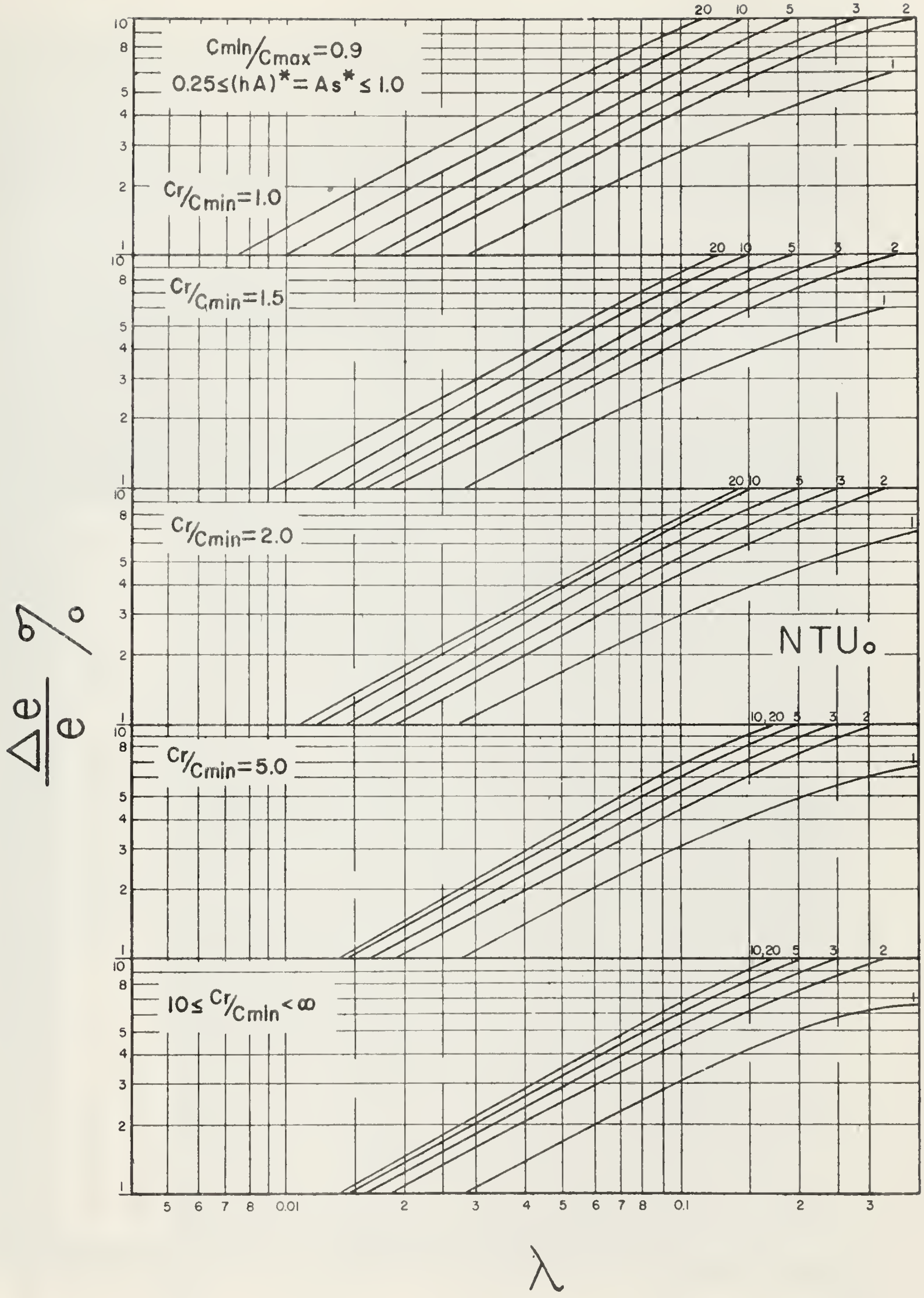

Cmin/Cmax=0.9
0.25≤(hA)*=As*≤1.0
Cr/Cmin=1.0
Cr/Cmin=1.5
Cr/Cmin=2.0
Cr/Cmin=5.0
10≤Cr/Cmin<∞
NTUo
20 10 5 3 2 1
10,20 5 3 2 1
Δe/e %
λ
10 8 5 4 3 2 1
5 6 7 8 0.01 2 3 4 5 6 7 8 0.1 2 3

FIG. 4

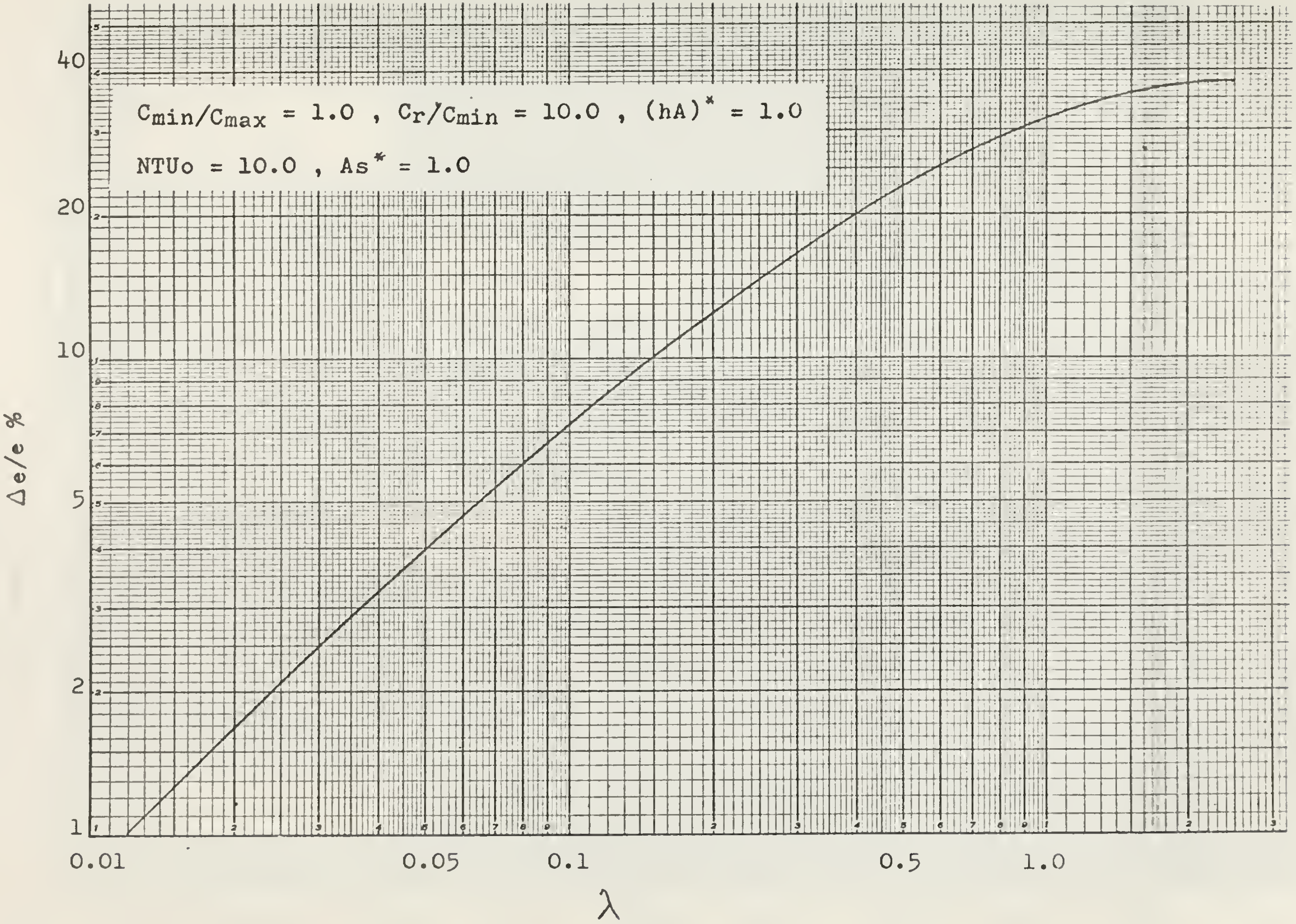

Cmin/Cmax = 1.0 , Cr/Cmin = 10.0 , (hA)* = 1.0
NTUo = 10.0 , As* = 1.0
Δe/e %
40
20
10
5
2
1
0.01
0.05
0.1
0.5
1.0
λ

Fig. 5

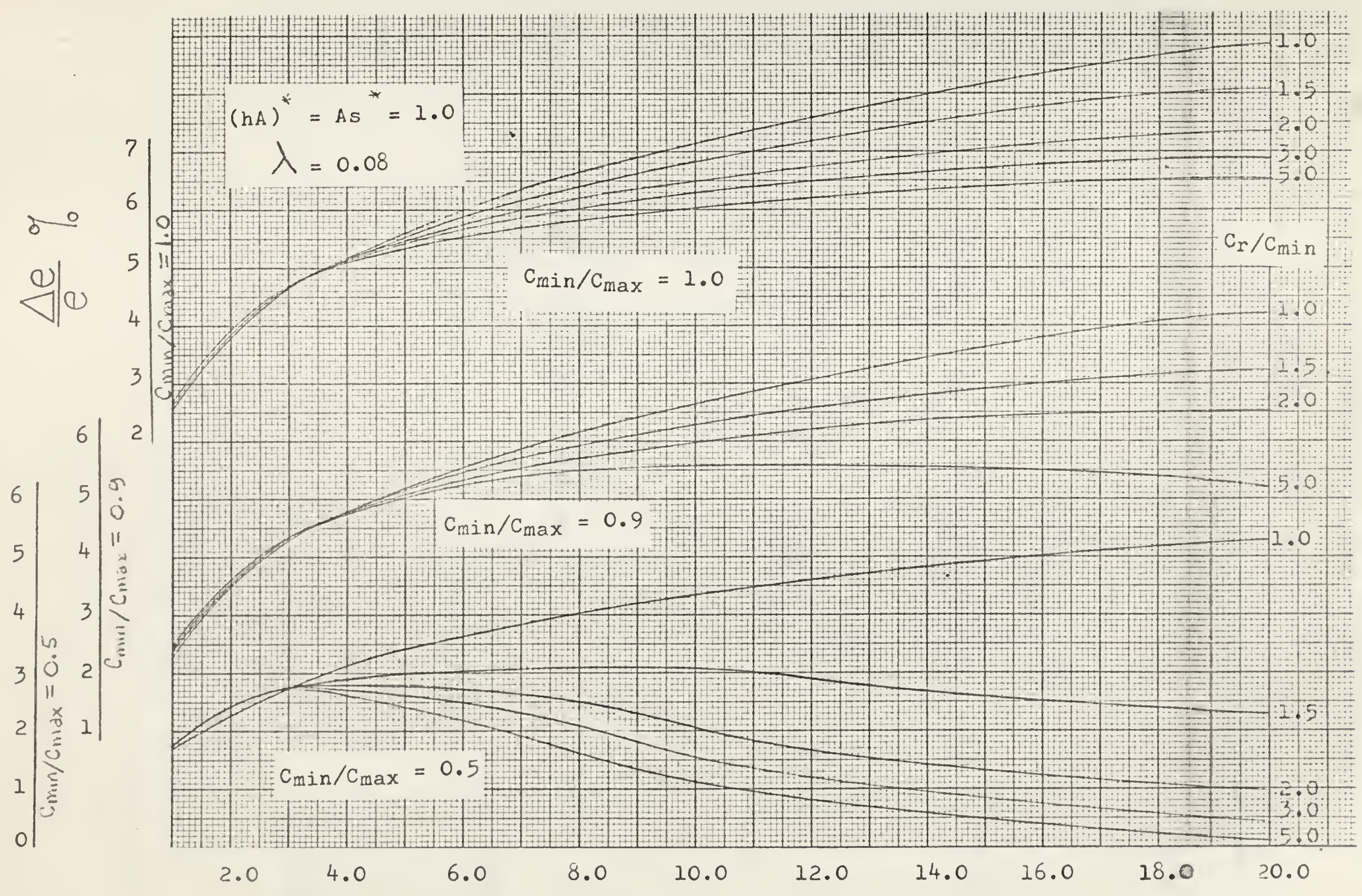

$(hA)^* = As^* = 1.0$
$\lambda = 0.08$
$\frac{\Delta e}{e}$ %
C_r/C_{min}
$C_{min}/C_{max} = 1.0$
$C_{min}/C_{max} = 0.9$
$C_{min}/C_{max} = 0.5$
1.0
1.5
2.0
3.0
5.0
1.0
1.5
2.0
5.0
1.0
1.5
2.0
3.0
5.0
7
6
5
4
3
2
6
5
4
3
2
1
6
5
4
3
2
1
0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0
NTUo

Fig. 6

APPENDIX 1.

Derivation of iterative equations.-

Equations (1) and (2) from page 5 can be put in the following form:

$$t_{x(i,j)} - t_{x(i+1,j)} = C_1\left[t_{x(i,j)} + t_{x(i+1,j)} - T_{x(i,j)} - T_{x(i,j+1)}\right] \qquad (1.1)$$

$$t_{x(i,j)} - t_{x(i+1,j)} =$$ [illegible] $$\left[T_{x(i,j+1)} - T_{x(i,j)}\right] - C_s\left[T_{x(i-1,j)} + T_{x(i-1,j+1)} - 2\left(T_{x(i,j)} + T_{x(i,j+1)}\right) + T_{x(i+1,j)} + T_{x(i+1,j+1)}\right], \qquad (1.2)$$

where

$C_1 =$ [illegible] NTU [illegible]

$C_s =$ [illegible]

and

[illegible] .

Equations (1.1) and (1.2) may be written as

[illegible] (1.3)

and

[illegible] (1.4)

[illegible] .

By adding to equation (1.3) the product of $(1+C_1)$(1.4) and rearranging

[illegible] (4)

where

[illegible]

[illegible] and [illegible] .

From equation (1.1)

[illegible] (6)

where

[illegible] and [illegible] .

For the elements from the first and last row equation (1.2) becomes

[illegible] (1.5)

and

[illegible] (1.6)

respectively. I[illegible] the same procedure as before and solving for the matrix outlet temperatures, the following equations are obtained:

[illegible] (3)

and

[illegible] (5)

for the first and last row elements respectively, where

[illegible]

[illegible] and $D_2 =$ [illegible] .

Equations (7) and (8) may be written as

[illegible] (1.7)

and

[illegible]

[illegible] (1.8)

Equation (1.8) reduces to

$$t_{n(N_r,g)} - t_{n(N_r+1,g)} = E_2\left[T_{n(N_r,g+1)} - T_{n(N_r,g)}\right] + E_3\left[T_{n(N_r,g)} + T_{n(N_r,g+1)} - T_{n(N_r-1,g)} - T_{n(N_r-1,g+1)}\right] \qquad (1.9)$$

and

[illegible] (1.10)

for the first and last row elements respectively, where

[illegible]

[illegible]

By comparison of equations (1.7) and (1.8) with equations (1.1) and (1.2) it follows that the form is the same and only the coefficients are different. The same holds true for equations (1.9) and (1.10) with equations (1.6) and (1.5) respectively. By substitution the following equations are obtained:

[illegible] (9)

[illegible] (10)

[illegible] (11)

for the first, middle and last row elements respectively and

[illegible] (12)

which is the same for all the elements, where

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

and

[illegible]

Convergence condition.-

Equation (1.2) can be put in the following form:

$$[illegible] \qquad (1.11)$$

In the energy balance equations, the side of C_{max} was assumed to be the hot side so that by the second law of thermodynamics

$$[illegible]$$

and from equation (1.11)

$$[illegible] \qquad (1.12)$$

Rearranging equation (1.12)

$$[illegible] \qquad (1.13)$$

Again by the second law of thermodynamics the matrix temperature variation in the direction of fluid flow is such that

$$[illegible] \qquad (1.14)$$

and

[illegible] (1.15)

so that

[illegible] (1.16)

Substituting equation (1.16) in equation (1.13)

[illegible] (1.17)

Three cases are investigated:

1.- If C_3 C_2 then equation (1.17) becomes

[illegible]

2.- If $C_3 = 0$ then equation (1.17) becomes

[illegible]

3.- If $C_2 - 2C_3 = 0$ then equation (1.17) becomes

[illegible]

From the above cases it can be seen that a sufficient condition for equation (2) to converge is that

[illegible]

Substituting the values for the constants and rearranging yields

[illegible]

For the first and last row elements the convergence criterion becomes

[illegible]

which for substitution of the parameters gives

[illegible]

This condition is less stringent than for the middle row elements.

Similarly, for the side of C_{min} the sufficient condition for the middle row elements becomes

[illegible]

which for substitution of the parameters gives

[illegible]

For the first and last row elements it is found that

[illegible]

which requires that

[illegible]

is the sufficient condition. This again is a less stringent condition than that imposed by the middle row elements.

Differential equations.-

Hausen [4] has given a theory for regenerators without heat conduction. When the same assumptions are made, but the longitudinal conduction of heat is included, (See page [illegible]) the differential equations obtained are [illegible] :

for the side of C_{max} (gas flowing from $y = 0$ to [illegible] $= L$);

$$[illegible]$$

$$[illegible]$$

and for the side of C_{min} (reversed gas flow),

$$[illegible]$$

$$[illegible]$$

A temperature scale can be used for which the gas entrance temperature at the side of C_{min} is zero and that at the side of C_{max} is unity. The boundary conditions are then

$$[illegible]$$

and

$$[illegible]$$

APPENDIX 2

Computer program.-

The Fortran system was used to program the problem for the CDC 1604 digital computer.

The six dimensionless parameters are used as input together with increments and factors to change the values of NTUo, and As, so that solutions can be generated for several combinations of the parameters without having to feed into the computer these different sets of parameters every time.

Another input is the number of subdivisions of the three streams together with the increments, so that when the computer finishes the first series of runs for a given number of subdivisions it automatically increases the subdivisions by the specified increments and the two values of effectiveness obtained are extrapolated to an infinite number of subdivisions.

In order to obtain the initial estimates of matrix temperatures, the problem is first solved neglecting the longitudinal heat conduction as explained previously. For this case the equations derived in Appendix 1 are simplified to (for detailed derivation see Reference 1)

[illegible]

[illegible]

[illegible]

[illegible]

where

$$A_1 = [illegible]^{-1}$$

$$A_2 = [illegible]^{-1}$$

$$A_3 = [illegible]^{-1}$$

$$A_4 = [illegible]^{-1}$$

The inlet matrix temperatures ([illegible]) necessary to initiate the problem for zero heat conduction are estimated by assuming [illegible], (See Fig. 1[illegible]) and feeding this value into the computer. The remaining inlet matrix temperatures are obtained from the relation

$$\bar{T}_{x(i,1)} = \left[1 - \frac{[illegible]}{N} \right] \bar{T}_{[illegible]}$$

The number of passes necessary to meet the desired accuracy in the solution changes accordingly with the initial estimates of matrix inlet temperatures, but considering that the computer time required to work the problem for zero heat conduction is so small as compared to the case with conduction, that these estimates do not become important. Only for high values of C_r/C_{min} these estimates may be of significance since for this case the change in the reversal condition after every pass is very small.

In general, it was found that an estimate $T_{(1,1)}$

of 0.4 to 0.6 for $C_r/C_{min} < 5.0$

and 0.7 to 0.9 for $C_r/C_{min} > 5.0$

together with the above relation for $T_{X(i,1)}$ should converge to the solution in a reasonable time.

The temperature distribution is calculated by repetitive use of the iteration equations.

The effectiveness may be defined as

$$\epsilon = \frac{C_{max}(t_{xi} - t_{xo})}{C_{min}(t_{xi} - t_{ni})} = \frac{t_{no} - t_{ni}}{t_{xi} - t_{ni}} .$$

from the conditions of the problem

$$T_{x_i} = \frac{1}{N_x} \sum_{1}^{N_x} t_{x(1,j)} = 1$$

$$T_{x_o} = \frac{1}{N_x} \sum_{1}^{N_x} t_{x(N_x+1,j)}$$

$$t_{n_i} = \frac{1}{N_n} \sum_{1}^{N_n} t_{n(1,j)} = 0$$

$$t_{n_o} = \frac{1}{N_n} \sum_{j=1}^{N_n} t_{n(N_x+1,j)}$$

therefore, the expression for effectiveness for the definition reduces to

$$\epsilon = \frac{1}{N_n} \sum_{j=1}^{N_n} t_{n(N_x+1,j)}$$

which is the expression used for the computation of the effectiveness.

The heat balance error is computed from the relation

$$\text{error} = \frac{Q_x - Q_n}{Q_x} = 1 - \frac{C_{min}(t_{n_o} - t_{n_i})}{C_{max}(T_{x_i} - T_{x_o})}$$

which reduces to

$$\text{error} = 1 - \frac{C_{min}}{C_{max}} \frac{\epsilon}{1 - \sum_{1}^{N_x} t_{x(N_x+1,j)}}$$

The extrapolated effectiveness is computed from the relation (see Fig. 2)

$$\epsilon_\infty = (1 + RE)\,\epsilon_2 - RE\,\epsilon_1$$

where

$$RE = \frac{N_1(N_2 + 1)}{N_2(N_1 + 1) - N_1(1 + N_2)} ,$$

The input parameters in Fortran notation are

$P1 = C_{min}/C_{max}$, $P2 = C_r/C_{min}$, $P3 = (hA)^*$,

$P4 = NTUo$, $P5 = \lambda$, $P6 = As^*$

The initial values of NTUo, λ and As* are changed by the following relations

```
P4 = P4 + P4IN
P5 = P5*P5FA + P5IN
P6 = P6 - P6IN
```

where P4IN, P5FA, P5IN, P6IN are also input values. The inputs P4FI, P5FI and P6FI are the final values of P4, P5, and P6 respectively.

The program is written so that the computer after working the initial set of parameters, it reduces P6 by P6IN every time until P6 becomes less than P6FI. Then it sets P6 equal to the initial value and changes P5. The procedure is again repeated until P5 becomes greater than P5FI. Then P5 is set equal to the initial value and P4 is increased by P4IN. The cycle is repeated until P4 becomes equal to or greater than P4FI.

During the computation, provisions are made by the use of Pause Statements and switches to make branch decisions in case the solution diverges or does not converge within the maximum number of passes specified. Figs. 7a and 7b show the complete flow diagram of the program which is followed by the Fortan listing of the program.

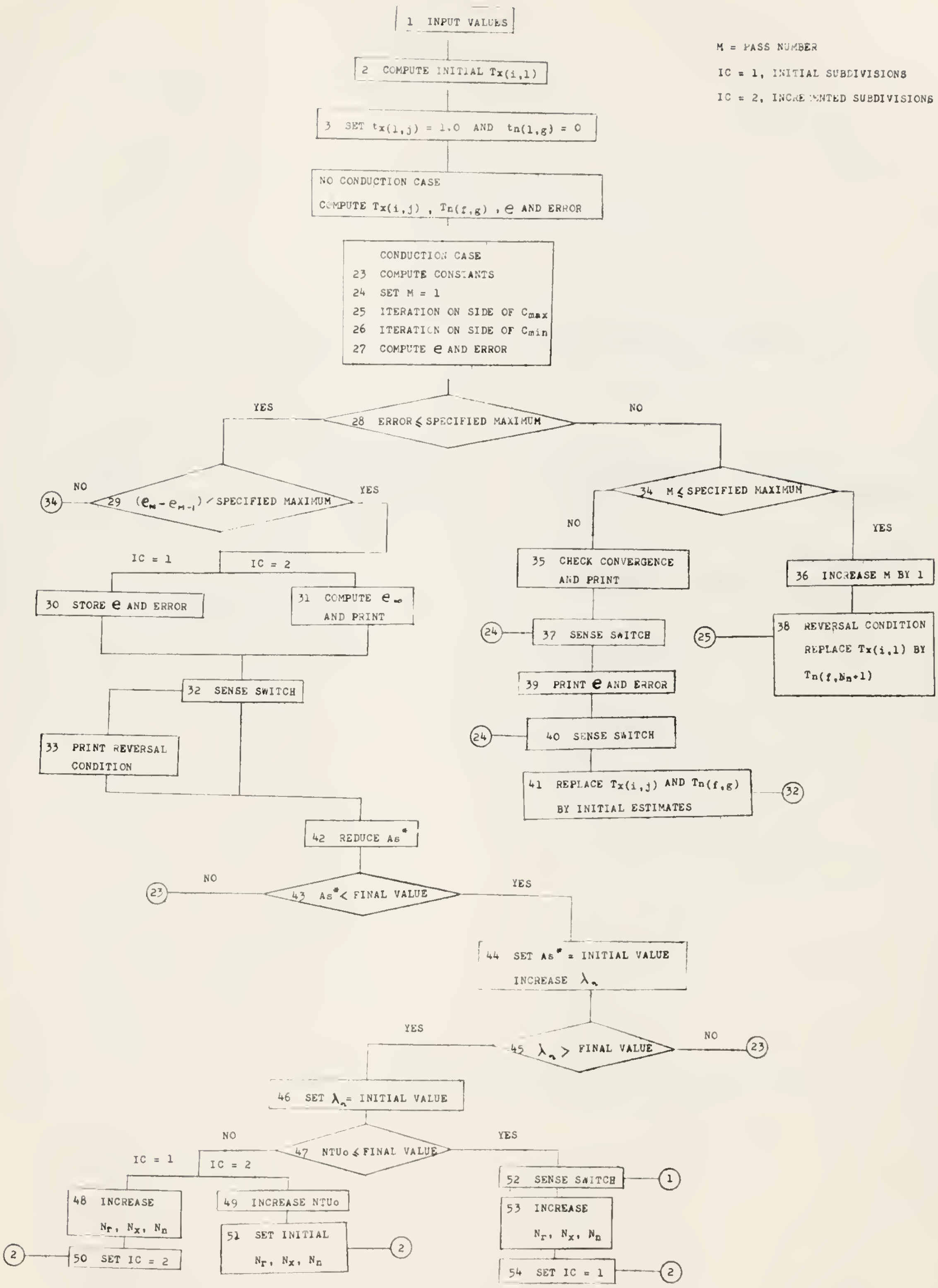
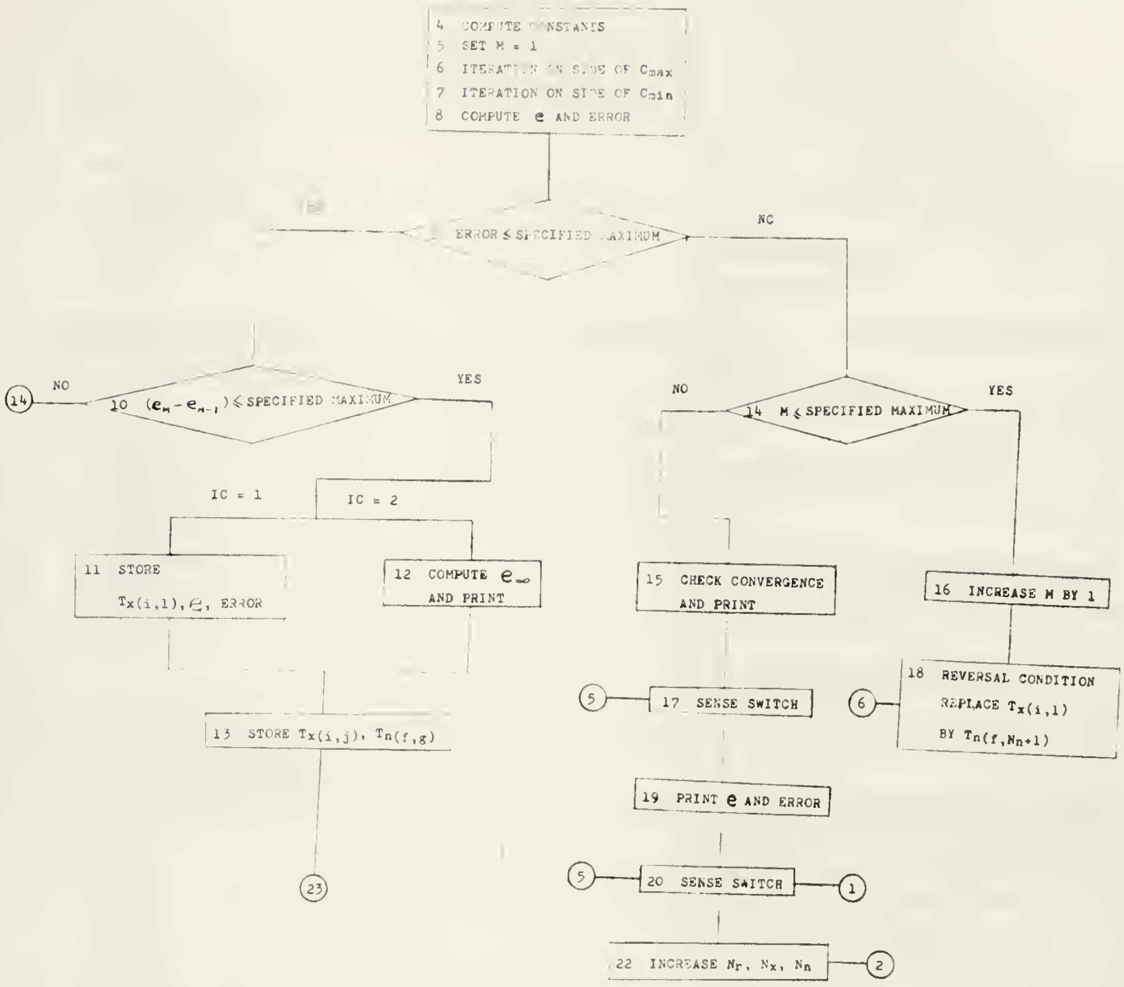

1 INPUT VALUES
2 COMPUTE INITIAL Tx(i,1)
3 SET tx(1,j) = 1.0 AND tn(1,g) = 0
NO CONDUCTION CASE
COMPUTE Tx(i,j) , Tn(f,g) , e AND ERROR
CONDUCTION CASE
23 COMPUTE CONSTANTS
24 SET M = 1
25 ITERATION ON SIDE OF Cmax
26 ITERATION ON SIDE OF Cmin
27 COMPUTE e AND ERROR
M = PASS NUMBER
IC = 1, INITIAL SUBDIVISIONS
IC = 2, INCREMENTED SUBDIVISIONS
YES
28 ERROR ≤ SPECIFIED MAXIMUM
NO
NO
34
29 (eM - eM-1) < SPECIFIED MAXIMUM
YES
34 M ≤ SPECIFIED MAXIMUM
NO
YES
IC = 1
IC = 2
30 STORE e AND ERROR
31 COMPUTE e∞ AND PRINT
35 CHECK CONVERGENCE AND PRINT
36 INCREASE M BY 1
24
37 SENSE SWITCH
25
38 REVERSAL CONDITION REPLACE Tx(i,1) BY Tn(f,Nn+1)
32 SENSE SWITCH
39 PRINT e AND ERROR
33 PRINT REVERSAL CONDITION
24
40 SENSE SWITCH
41 REPLACE Tx(i,j) AND Tn(f,g) BY INITIAL ESTIMATES
32
42 REDUCE As*
NO
23
43 As* < FINAL VALUE
YES
44 SET As* = INITIAL VALUE INCREASE λn
YES
45 λn > FINAL VALUE
NO
23
46 SET λn = INITIAL VALUE
NO
47 NTUo ≤ FINAL VALUE
YES
IC = 1
IC = 2
52 SENSE SWITCH
1
48 INCREASE Nr, Nx, Nn
49 INCREASE NTUo
53 INCREASE Nr, Nx, Nn
51 SET INITIAL Nr, Nx, Nn
2
2
50 SET IC = 2
54 SET IC = 1
2

FIG. 7a GENERAL FLOW DIAGRAM



4 COMPUTE CONSTANTS
5 SET M = 1
6 ITERATION ON SIDE OF C_{max}
7 ITERATION ON SIDE OF C_{min}
8 COMPUTE e AND ERROR
YES
ERROR ≤ SPECIFIED MAXIMUM
NO
14
NO
10 $(e_M - e_{M-1}) \leqslant$ SPECIFIED MAXIMUM
YES
IC = 1
IC = 2
11 STORE $T_x(i,1)$, e, ERROR
12 COMPUTE e_∞ AND PRINT
13 STORE $T_x(i,j)$, $T_n(f,g)$
23
NO
14 M ≤ SPECIFIED MAXIMUM
YES
15 CHECK CONVERGENCE AND PRINT
16 INCREASE M BY 1
5
17 SENSE SWITCH
6
18 REVERSAL CONDITION REPLACE $T_x(i,1)$ BY $T_n(f,N_n+1)$
19 PRINT e AND ERROR
5
20 SENSE SWITCH
1
22 INCREASE N_r, N_x, N_n
2

FIG. 7b FLOW DIAGRAM FOR NO CONDUCTION

PROGRAM | REMARKS

```
      PROGRAM BAHNKE
      OIMENSION TH(40,40),UH(40,401,TC(40,401,UC(40,40),ERR(92),TCO(92),
     1UHC(40,401,UCC(40,40),UH1(401,UH2(40),EF(20,20),FR!20,201
```

```
)     FORMAT(6F5.21
      REAO 1,P1,P2,P3,P4,P41N,P4F1
2     FORMAT(6151
      READ 2,NM,NH,NC,NM1,NH(,NC1
```

INPUT NM = N_r, NH = N_x, NC = N_n

```
3     FORMAT(F5.3)
      REAO 3,UH(1,11
      EM=NM
      UO 401 K=1,NM
      E=K
401   UH(K,11=((EM+1.0-E1*UH(1,1)1/EM
```

INPUT: UH(1,1) = $T_x(1,1)$

ESTIMATES OF $T_x(1,1)$

```
4     FORMAT(7F7.21
      REAO 4,P5,P5IN,P5FA,P5F1,P6,P61N,P6F1
```

INPUT

```
      S5=P5
      S6=P6
```

STORE INITIAL P5 AND P6

```
5     NM2=NM+NM1
      NH2=NH+NH1
      NC2=NC+NC1
```

NUMBER OF SUBDIVISIONS FOR 2nd RUN

```
      NE1=NM*(NH+NC1
      RE1=NE(
      NE2=NM2*(NH2+NC2)
      RE2=NE2
      RE=RE1/(RE2-RE1)
```

RE = VARIABLE USED TO EXTRAPOLATE EFFECTIVENESS

```
      EM2=NM2
      DO6 L=1,NM2
      E=L
      RT=1.0-((E-1.0)/EM2)
6     UH2(L1=UH(1,11*RT
```

ESTIMATES OF $T_x(1,1)$ FOR 2nd RUN

```
7     FORMAT(//14HNO OF ELEMENTS3X,3I5,4X,3(5)
      WRITE OUTPUT TAPE 2,7,NM,NH,NC,NM2,NH2,NC2
      PR(NT 7,NM,NH,NC,NM2,NH2,NC2
```

```
8     IC=1
```

INDICATES 1st RUN, USING INITIAL No OF SUBDIV.

```
9     EM=NM
      EH=NH
      EC=NC
```

FIXED TO FLOATING POINT CONVERSION

```
      NMM=NM-(
      NHP=NH+(
      NCP=NC+1
      NMP=NM+1
```

DEFINITION OF NEW VARIABLES

```
      OO 10 J=1,NH
10    TH(1,J1=1.0
      DO 11 L=1,NC
1)    TC(1,L1=0.0
```

SET $t_{x(1,j)}$ = 1.0 AND $t_n(1,g)$ = 0.0

```
)2    A10=(.0+EM/(P1*P2*EH1+(2.0*EM*P31/(P4*P1*().0+P31)
      A1=2.0/A10
      A20=1.0+(P1*P2*EH1/EM+(2.0*EH*P3*P21/(P4*(1.0+P311
      A2=2.0/A20
      A30=1.Q+EM/(P2*EC1 +(2.0*EM)/(P4*(1.0+P311
      A3=2.0/A30
      A40=1.0+(P2*EC1/EM +(2.0*EC*P21/(P4*().0+P31)
      A4=2.0/A40
```

CONSTANTS FOR THE NO CONDUCTION CASE

```
C     HOT SIOE NO CONDUCT)ON
```

```
121   M=1
```

M DENOTES PASS NUMBER

```
13    OO )5 J=1,NH
      DO 15 I=1,NM
      TH(1+(,J)=TH(1,J1+A1*(UH(1,J1-TH(1,J11
15    UH(I,J+1)=UH(I,J1-A2*(UH(I,J1-TH(1,J1)
```

ITERATION ON SIDE OF C_{max}

```
C     COLO SIDE NO CONDUCTION
      DO 17 K=1, NM
17    UC(K,11=UH(NM+1-K,NH+11
      DO 19 L=1,NC
      DO19 K=(,NM
      TC(K+1,L)=TC(K,L)+A3*(UC(K,L)-TC(K,L1)
)9    UC(K,L+11=UC(K,L1-A4*(UC(K,L)-TC(K,L))
```

ITERATION ON SIDE OF C_{min}

```
      G1=0.0
      OO 20 J=1,NH
20    G1=TH(NM+1,J1+G1
      THO=G1/EH
```

THO = $\left[t_{x(N_r+1,j)}\right]$ avg

```
      G2=0.0
      DO 21 L=1,NC
21    G2=TC(NM+1,L1+G2
      TCO(M)=G2/EC
```

TCC = EFFECTIVENESS

```
      ERR(M1=1.0-(P1*TCO(M1)/(1.0-THO)
      IF(ERR(M) )23,24,24
23    ERR(M1=-ERR(M)
24    IE(ERR(M)-4.0F-61 30,30,25
```

COMPUTE HEAT BALANCE ERROR AND COMPARE IT WITH MAXIMUM SPECIFIED

```
25    (F(90-M1 26,26,215
```

```
26    IF(ERR(M-1)-ERR(M)) 29,29,27
27    PAUSE 11
28    IF(SENSE SWITCH 11 32,121
29    PAUSE I2
      GO TO 28
```

CHECK CONVERGENCE

PAUSE 11 = SOLUTION CONVERGES

PAUSE 12 = SOLUTION DIVERGES

```
215   DO 22 I=1,NM
22    UH(I,1)=UC(NM+1-I,NC+1)
      M=M+1
      GO TO 13
```

REVERSAL CONDITION

```
30    T2=TCO(M-I)-TCO(M)
      IF(T2) 301,302,302
301   T2=-T2
302   IF(T2-4.0E-61 305,305,25
```

CHECK DIFFERENCE IN EFFECTIVENESS IN THE LAST TWO PASSES

```
305   IF((C-1) 31,31,33
31    DO 312 J=1,NM
312   UH1(J)=UH(J,11
      EF(1,1)=TCO(M1
      ER(1,()=ERR(M)
      GO TO 39
```

STORE $T_{x(i,1)}$, EFFECTIVENESS AND ERROR FOR THE 1st RUN

```
32    FORMAT(4F8.2,I5,F10.6,F9.2,I3)
      PRINT 32,P1,P2,P3,P4,M,TCO(M),ERR(M),IC
      PAUSE 13
      IF(SENSE SWITCH 1) 321,121
321   IF(SENSE SWITCH 2) 1,322
```

```
322   NM=NM2
      NH=NH2
      NC=NC2
      GO TO 5
```

INCREASE NO. OF SUBDIVISIONS AND REPEAT 1st RUN

```
33    EFF=TCO(M)*(1.0+RE)-EF(1,1)*RE
      B1=EFF
```

EXTRAPOLATE EFFECTIVENESS AND STORE

```
34    FORMAT(//4F8.2,16X,2(F10.6,F9.2),F10.6)
      WRITE OUTPUT TAPE 2,34,P1,P2,P3,P4,EF(1,1),ER(1,1),TCO(M),ERR(M),
     1EFF
```

```
      DO 35 L=1,NM
35    UH2(L)=UH(L,1)
```

STORE $T_{x(i,1)}$ FROM 2nd RUN

```
39    IA=2
      IB=1
```

SUBSCRIPTS FOR ERROR AND EFFECTIVENESS

```
40    DO 44 I=1,NM
41    DO 42 J=1,NHP
42    UHC(I,J1=UH(I,J)
43    DO 44 L=I,NCP
44    UCC((,L)=UC(I,L1
```

UHC(I,J) = $T_{x(i,j)}$, UCC(I,L) = $T_{n(f,g)}$

FOR CONDUCTION CASE

```
C     CONSTANTS
50    E1=(P4*(1.0+P3))/(2.0*EM)
      E11=1.0+E1
      E2=(P2*EC)/EM
      E3=0.5*P5*EM
      E4=E1+(E11*(2.0*E3+E21)
      E5=E1+(E11*(E2+E3))
      F1=(2.0*E11/E4
      F2=(E1+(E11*(2.0*E3-E2)))/E4
      F3=(E3*E11)/E4
      F4=(1.0-E11/E11
      F5=E1/E11
      F6=(2.0*E11/E5
      F7=(E1+(E11*(E3-E2)11/E5
      F8=(E3*E11)/E5
      C1=(P1*P4*(1.0+P31)/(2.0*P3*EM)
      C11=1.0+C1
      C2=(P1*P2*EH)/EM
51    C3=(P1*P5*EM1/(2.0*P6)
      C4=C1+(C11*(C2+2.0*C3))
      C5=C1+(C11*(C2+C3))
```

CONSTANTS FOR CONDUCTION CASE

```
      D1=(2.0•C1)/C4
      D2=(C1+(C11•(2.D•C3-C2)))/C4
      03=(C3•C11)/C4
      D4=(1.0-C1)/C)1
      05=C1/C11                                                      CONSTANTS FOR CONDUCTION CASE
      D6=(2.0•C11/C5
      07=(C1+(C11•(C3-C2)))/C5
      08=(C3•C111/C5
 53   M=1
C     HOT SIDE WITH CONDUCTION
 60   OD 67 J=),NH
      L=)
 6)   UHC(I,J+))=D6•TH(1,J)-07•UHC(),J)+D8•(UHC(2,J)+UHC(2,J+1))
      TH(2,J)=04•TH(1,J)+D5•(UHC(1,J)+UHC(1,J+)))
 62   D064 I=2,NMM
      UHC(),J+1)=D)•TH(),J)-02•UHC(1,J)+03•(UHC(1-1,J)+UHC()-),J+1)+  ITERATION ON SIDE OF Cmax
     1UHC(1+),J)+UHC(1+1,J+1))                                       TWO PASSES PER COLUMN
 64   TH()+1,J)=04•TH(1,J)+D5•(UHC(1,J)+UHC(I,J+)))
      UHC(NM,J+1)=06•TH(NM,J)-C7•UHC(NM,J)+D8•(UHC(NMM,J)+UHC(NMM,J+1))
 66   TH(NMP,J)=04•TH(NM,J)+D5•(UHC(NM,J)+UHC(NM,J+1))
      L=L+1
      )F(3-L) 67,67,61
 67   CONTINUE
 68   DO 69 K=),NM
 69   UCC1K,1)=UHC(NMP-K,NHP)
C     COLO SIDE WITH CONOUCTION
 70   DO 75 L=1,NC
      J=1
 71   UCC(1,L+1)=F6•TC(),L)-F7•UCC(),L)+F8•(UCC(2,L)+UCC(2,L+1)1
      TC(2,L)=F4•TC(1,L)+F5•(UCC(1,L)+UCC(),L+1))
 72   DO 73 K=2,NMM
      UCC(K,L+1)=F1•TC(K,L)-F2•UCC(K,L)+F3•(UCC(K-1,L)+UCC(K-1,L+1)+  ITERATION ON SIDE OF Cmin
     )UCC(K+1,L)+UCC(K+1,L+1))                                       TWO PASSES PER COLUMN
 73   TC(K+),L)=F4•TC(K,L)+F5•(UCC(K,L)+UCC(K,L+)))
      UCC(NM,L+1)=F6•TC(NM,L)-F7•UCC(NM,L)+F8•(UCC(NMM,L)+UCC(NMM,L+1))
 74   TC(NMP,L)=F4•TC(NM,L)+F5•(UCC(NM,L)+UCC(NM,L+1))
      J=J+1
      1F(3-J) 75,75,71
 75   CONTINUE
      G1=0.D
      DO 76 J=1,NH
 76   G1=TH(NMP,J1+G1
      THO=G1/EH
      G2=0.0
      OO 78 L=1,NC
 78   G2=TC(NMP,L)+G2
      TCO(M)=G2/FC
      ERR(M)=).0-(P)•TCO(M))/(1.0-THO)
      )F(ERR(M)) 80,80,81
 80   ERR1M)=-ERR(M)
 81   1F(ERR(M)-4.0E-6) 90,90,E2
 82   IF(90-M) 83,83,85
 83   IF(ERR(M-1)-ERR(M)) 84,84,87
 84   PAUSE 22
 841  IF(SENSE SWITCH 1) 842,53
 842  IF(SENSE SWITCH 2) 88,884
 85   M=M+1
      OO 86 I=1,NM
 86   UHC(I,I)=UCC(NMP-I,NCP)
      GO TO 60
 87   PAUSE 21
      GO TO 841
 88   1F(IC-)) 881,881,882
 881  EF(IA,181=0.D
      ER(IA,18)=0.0
      GO TO 97
 882  FORMAT(11HSOL DIV FOR13,18X,E9.2,F8.2,2(F10.6,F9.2))
      WRITE OUTPUT TAPE 2,882,IC,P5,P6,FF(IA,18),ER(IA,IB),TCO(M),ERR(M)
      GO TO 97
```

```
884  FORMAT(4F8.2,E8.2,F8.2,2X,I3,F10.6,E9.2)
     PRINT 884,P1,P2,P3,P4,P5,P6,IC,TCO(M),ERR(M)
     PAUSE 23
     IF(SENSE SWITCH 1) 885,53
885  IF(SENSE SWITCH 2) 114,88
89   IF(IC-1) 891,891,93
891  EF(IA,IB)=TCO(M)
     ER(IA,IB)=ERR(M)
     GO TO 102
90   T1=TCO(M-1)-TCO(M)
     IF(T1) 91,92,92
91   T1=-T1
92   IF(T1-4.0E-6) 89,89,82
93   EFF=TCO(M)*(1.0+RE)-EF(IA,IB)*RE
     82=(IB1-EFF)*100.0)/B1
94   FORMAT(32X,E8.2,F8.2,2(F10.6,E9.2),F10.6,F8.3)
     WRITE OUTPUT TAPE 2,94,P5,P6,EF(IA,IB),ER(IA,IB),TCO(M),ERR(M),EFF
    1,82
     IF(SENSE SWITCH 3) 95,102
95   FORMAT(F8.2,E9.2,F8.4)
     PRINT 95,P4,P5,82
     GO TO 102
97   00 974 I=),NM
971  00 972 J=1,NHP
972  UHC(1,J)=UH(I,J)
973  DO 974 L=1,NCP
974  UCC(1,L)=UC(1,L)
102  IF(SENSE SWITCH 3) 131,132
131  FORMAT(/4F8.2,E8.2,F8.2,3X,I3/(15F7.4))
     WRITE OUTPUT TAPE 3,131,P1,P2,P3,P4,P5,P6,IC,
    1(UHC(J,1),UCC(NMP-J,NCP),J=1,NM)
132  P6=P6-P6IN
     IB=IB+1
     IF(P6-P6FI) 103,103,51
103  P6=S6
     IB=1
104  P5=P5*P5FA+P5IN
     IA=IA+)
     IF(P5FI-P5) 105,50,50
105  P5=S5
     IF(IC-1) 106,106,108
106  IC=2
     NM=NM2
     NH=NH2
     NC=NC2
     DO 107 K=1,NM
107  UH(K,1)=UH2(K)
     GO TO 9
108  IF(P4FI-P4) 113,113,109
109  P4=P4+P4IN

     NM=NM-NM1
     NH=NH-NH1
     NC=NC-NC1
     00 111 K=1,NM
111  UH(K,1)=UH1(K)
     GO TO 8
113  PAUSE 50
     IF(SENSE SWITCH 1) 114,1
114  DO 115 L=1,NM
115  UH(L,1)=UH2(L)
     GO TO 5
     ENO
     ENO
```

UNCLASSIFIED

[redacted]

UNCLASSIFIED

thesB137
Effect of longitudinal heat conduction o
3 2768 001 91155 5
DUDLEY KNOX LIBRARY